Trigonometry - Functions and Graphs Lesson #1:
Angular Measure - Degrees

Overview

In this unit, we consider angles in standard position expressed in degrees and radians, we develop and apply the equation of the unit circle, we solve problems using trigonometric ratios, and we graph and analyze sine, cosine, and tangent functions.

Note that some of the work in this lesson is a review of work covered in previous courses.

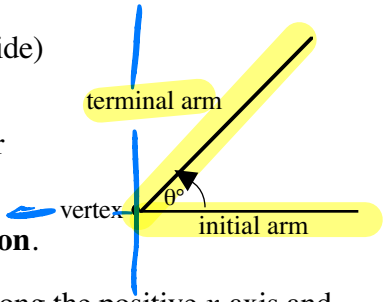
Rotation Angles in Standard Position

Angles can be measured in degrees where 360° is one complete rotation.

A **rotation angle** is formed by rotating an initial arm (or initial side) through an angle θ° about a fixed point (the vertex).

The angle formed between the initial arm and the terminal arm (or terminal side) is the rotation angle.

The angle shown in the diagram is said to be in **standard position**.

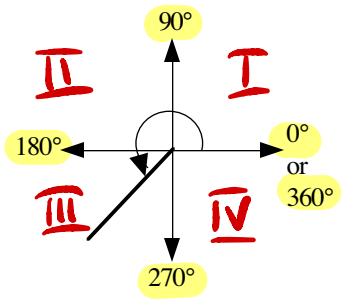


On a coordinate grid, standard position means the initial arm is along the positive x-axis and the rotation is about the origin.

CCW

A **positive angle** results from a counter clockwise rotation.

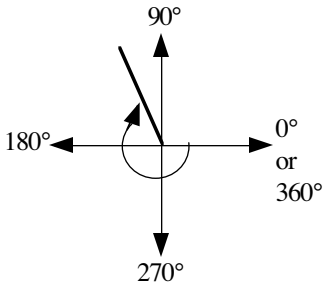
The diagram below shows an angle of 235° in standard position.



CW

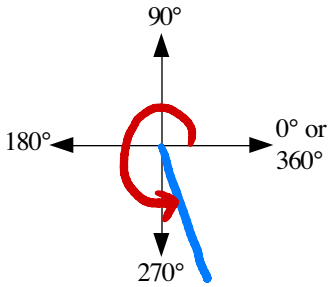
A **negative angle** results from a clockwise rotation.

The diagram below shows an angle of –247° in standard position.



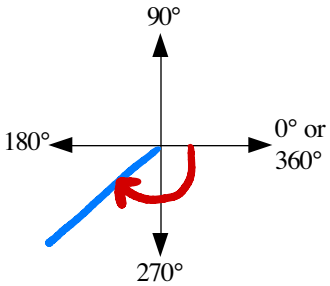
Sketch the rotation angle in standard position and state the quadrant in which the angle terminates.

a) 290°



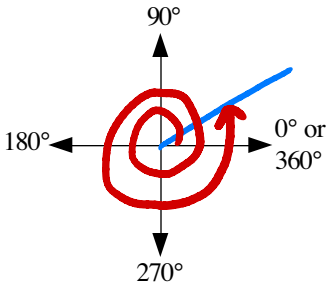
IV

b) –135°

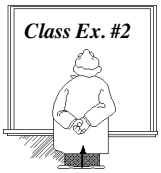


III

c) 750°

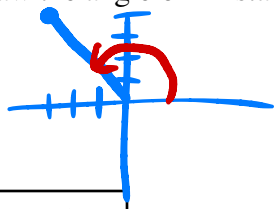


I



The point A lies on the terminal arm of the rotation angle $\theta°$.
In each case, draw the angle $\theta°$ in standard position.

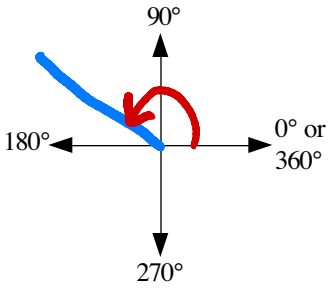
a) $A(-3, 4)$ **b)** $P(-7, -2)$



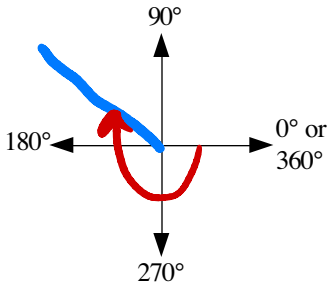
Coterminal Angles

a) Draw the rotation angle in standard position.

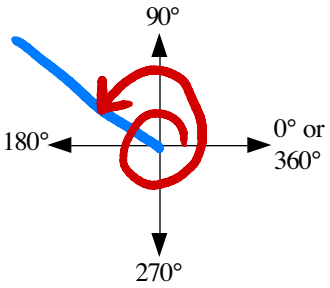
i) rotation of 150°



ii) rotation of –210°



iii) rotation of 510°



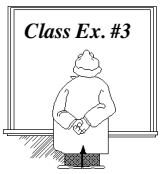
The three rotation angles above have the same terminal arm.
They are called **coterminal angles**.

b) The **principal angle** of a set of coterminal angles is the smallest positive rotation angle with the same terminal arm. The principal angle is always between 0° and 360°.

In the diagrams above, the principal angle is 150°.

c) There are infinitely many angles that are coterminal with a given angle. They can be found by adding or subtracting multiples of 360° to/from the given angle.
The set of all coterminal angles can be described with reference to the principal angle $P°$ as $(P + 360n)°$ where n is an integer.

The set of angles coterminal with the angles in the diagram can be written in the form $150° + 360n$ ("n" must be an integer)



In each case **i)** determine the angle(s), θ, in the domain $-360° \le \theta \le 360°$, which is coterminal with the given angle
ii) write an expression involving the principal angle that represents all angles in the domain $\theta \in R$ that are coterminal with the given angle

a) 285° $285 - 360° = -75°$

b) –13° $-13 + 360 = 347°$

c) 395° $395° - 360° = 35°$ $35° - 360° = -325°$

Complete Assignment Question #1 - #3

$285° + 360n$ n ∈ I $347° + 360n$ $35 + 360n$

always ∠ measured from the x-axis

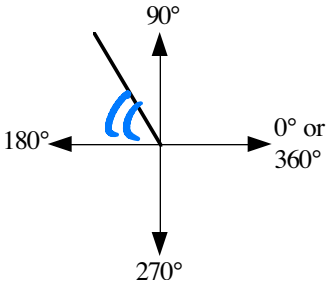
Reference Angles

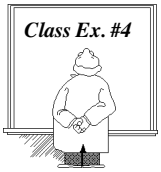
→ always +ive.
→ always acute

A **reference angle** is the acute angle formed between the terminal arm of the rotation angle and the x-axis.

The diagram shows the terminal arm of a rotation angle of 120° with a reference angle of 60°.

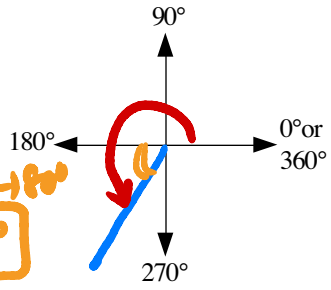
Mark 120° and 60° on the diagram.





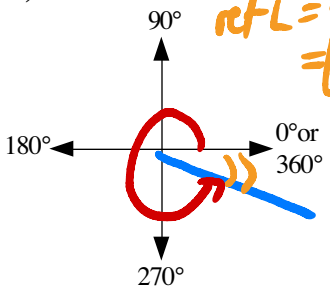
In each case, sketch the rotation angle and state the reference angle.

a) 243°



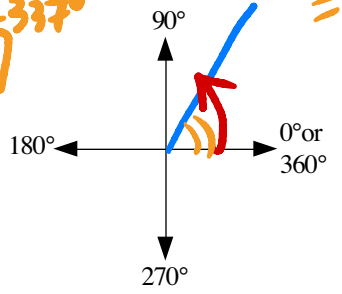
ref∠ = 243° − 180° = 63°

b) 337°



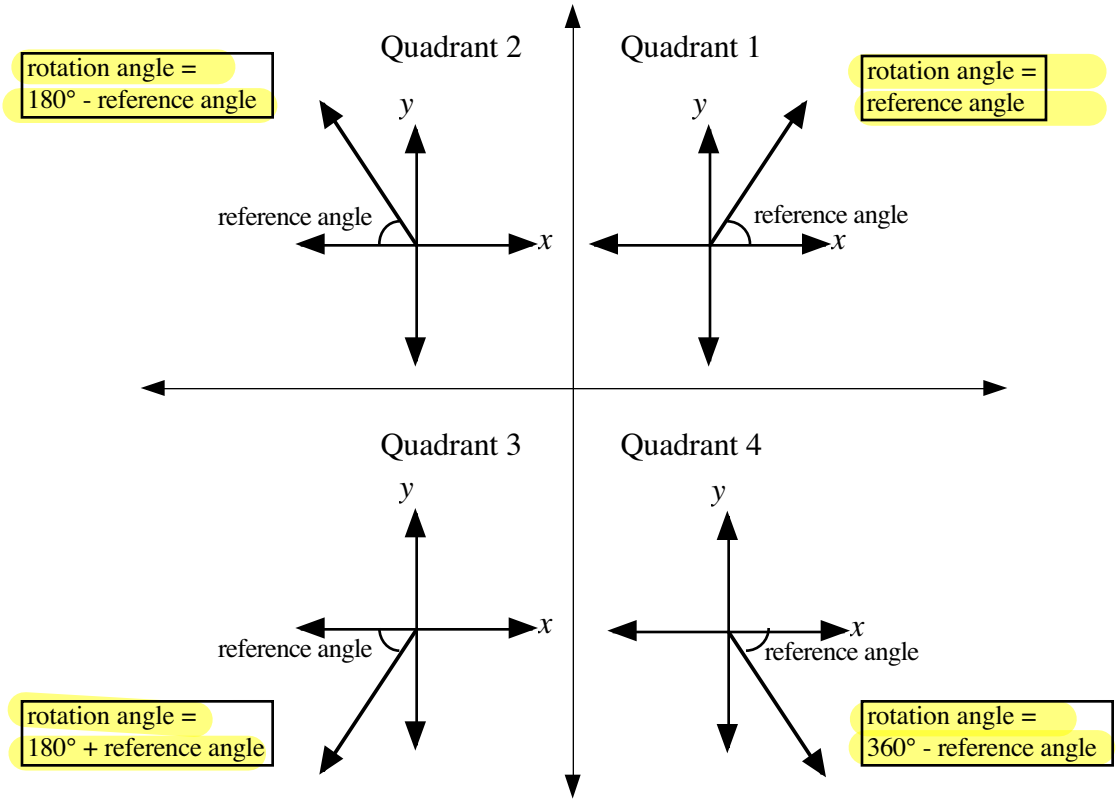
ref∠ = 360° − 337° = 23°

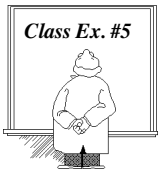
c) 70°



ref∠ = 70°

The diagram below describes the relationship between the reference angle and the rotation angle in each quadrant. If the rotation angle is not between and 0° and 360°, it needs to be converted to the principal angle for the relationship to be valid.





State the reference angle for each of the following rotation angles in standard position.

a) 195° Q 3 ref∠ = 15°

b) –258° +360° = 102° Q2 ref∠ = 78°

c) –810° Q ? → not in a quadrant it's on the y-axis ref∠ = 90°

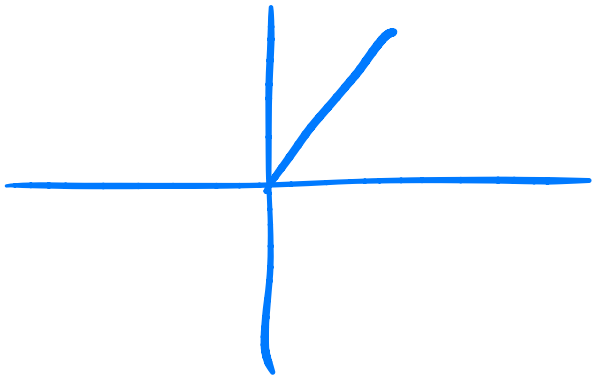
Determine four angles between 0° and 360° which have the same reference angle as a rotation angle of –128°.

I ref∠ = 52°

II 180 – 52° = 128°

III 180° + 52° = 232°

IV 360° – 52° = 308°



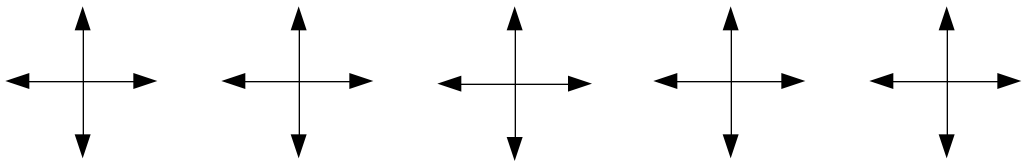
Complete Assignment Question #4 - #12

#1-12

Assignment

1. Sketch the following rotation angles in standard position, and state the quadrant in which the angle terminates.

a) 160° **b)** 318° **c)** –26° **d)** 569° **e)** –595°



2. Which of the following angles are coterminal with 80°?

i) 800° **ii)** –100° **iii)** –280° **iv)** 280°