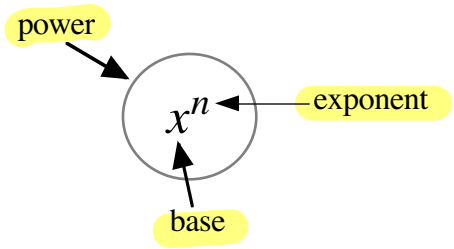
Exponential and Logarithmic Functions Lesson #1:
Review of Exponents

Overview

In this unit we will explain the relationship between logarithmic and exponential functions, and introduce the product, quotient, and power laws of logarithms.
We will graph and analyze exponential and logarithmic functions, and solve problems involving exponential and logarithmic equations.

Review of Exponent Laws

The exponent laws involve operations on powers.
The parts of a power are shown.



Complete the following exponent laws.

Product Law $x^m x^n =$ x^{m+n}

Quotient Law $x^m \div x^n =$ x^{m-n}

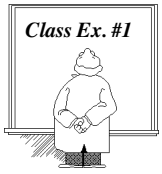
Power of a Power $(x^m)^n =$ x^{mn}

Power of a Product $(xy)^m =$ $x^m y^m$

Power of a Quotient $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, $y \neq 0$

Integral Exponent Rule $x^{-m} = \left(\frac{1}{x}\right)^m$, where $x \neq 0$

Rational Exponents $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$



Write each expression without brackets and with positive exponents.

a) $\frac{1}{2}y^{-6}$ $\frac{1}{2}\cdot\left(\frac{1}{y}\right)^6 = \frac{1}{2y^6}$

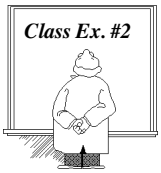
b) $\frac{5x^{-3}}{y^{-2}}$ $= \frac{5y^2}{x^3}$

c) $(4x^3y)(2x^{-4}y^2)$ $= 8x^{-1}y^3 = \frac{8y^3}{x}$

d) $\frac{24m^5p^{-3}q^4}{-4m^4p^2q^{-2}}$ $= \frac{-6mq^6}{p^5}$

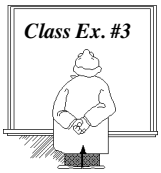
e) $(3x^2y^3)^3$ $= 3^3x^6y^9 = 27x^6y^9$

f) $\frac{12b^{-\frac{1}{2}}}{3b}$ $= 4b^{-\frac{3}{2}} = \frac{4}{b^{\frac{3}{2}}} = \frac{4}{\sqrt{b^3}}$



Simplify the following. Write the answers with positive exponents.

a) $(4xy^{-2})^{-3}$ **b)** $\left(\dfrac{3x^{3}}{4y^{-2}}\right)^{-2}$

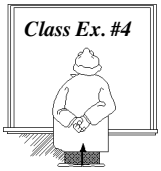


Without using a calculator, determine exact value of the following. Verify with a calculator.

a) 2^{-3} **b)** $81^{\frac{3}{4}}$ **c)** 25^{0} **d)** $16^{-\frac{1}{2}}$

Complete Assignment Questions #1 - #3

Changing Base



Convert each of the following to the base indicated.

a) 9^{2x} to base 3 $(3^2)^{2x} = 3^{4x}$

b) 125^{2-x} to base 5 $(5^3)^{2-x} = 5^{6-3x}$

c) $8 \cdot 16^{x}$ to base 2 $2^3 \cdot (2^4)^x = 2^3 \cdot 2^{4x} = 2^{3+4x}$

d) $\dfrac{1}{512^{3x}}$ to base 2 $512^{-3x} = (2^9)^{-3x} = 2^{-27x}$

e) $\left(\dfrac{16}{81}\right)^{x+5}$ to base $\dfrac{2}{3}$ $\left(\left(\dfrac{2}{3}\right)^4\right)^{x+5} = \left(\dfrac{2}{3}\right)^{4x+20}$

Complete Assignment Questions #4 - #5

Solving Equations with Rational Exponents

We have already met this concept in earlier courses. Consider the following example:

The volume of a beach ball is 50 965 cm^3. Determine the radius of the ball to the nearest tenth of a cm. $\left(\text{Volume of sphere} = \frac{4}{3}\pi r^3\right)$

Sara and Lee are solving the problem and the first four steps in their solutions are identical as shown below.

$$V_{Sphere} = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad 50965 = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad 3(50965) = 4\pi r^3 \quad \Rightarrow \quad \frac{3(50965)}{4\pi} = r^3$$

a) Sara completed the solution by taking the cube root of each side of the equation.

Complete Sara's solution.

$$r^3 = \frac{3(50965)}{4\pi}$$

$r =$

b) Lee completed the solution by raising each side of the equation to a specific power.

i) Which power did he use?

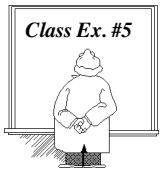
ii) Complete Lee's method.

$$r^3 = \frac{3(50965)}{4\pi}$$

$r =$

Use the following procedure to solve an equation where the exponent is rational:

- Raise both sides to the reciprocal power of the exponent
- Simplify and solve for the variable.



Solve for x in the following.

a) $x^{-\frac{4}{3}} = 81$

$\left(x^{1-\frac{4}{3}}\right)^{-\frac{3}{4}} = 81^{-\frac{3}{4}}$

$x = \frac{1}{81^{\frac{3}{4}}}$

$x = \frac{1}{\sqrt[4]{81^3}} = \boxed{\frac{1}{27}}$

b) $(3x-5)^{\frac{3}{2}} = 27$

$\left((3x-5)^{\frac{3}{2}}\right)^{\frac{2}{3}} = 27^{\frac{2}{3}}$

$3x-5 = \sqrt[3]{27^2}$

$3x-5 = 9$

$\frac{3x}{3} = \frac{14}{3}$

$x = \boxed{\frac{14}{3}}$

Complete Assignment Questions #6 - #9

#1-9