Exponential and Logarithmic Functions Lesson #2:
Solving Exponential Equations with a Common Base

Review

Simplify $(9^{2x+3} \div 27^{3x-1}) \times 81^{x-1}$ by converting each term to a common base.

$$\left((3^2)^{2x+3} \div (3^3)^{3x-1}\right) \times (3^4)^{x-1}$$

$$3^{4x+6} \div 3^{9x-3} \times 3^{4x-4}$$

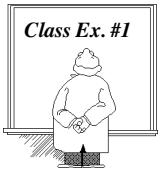
$$3^{-5x+9} \times 3^{4x-4} = \boxed{3^{-x+5}}$$

Solving Exponential Equations with a Common Base

An **exponential equation** is an equation where the variable is in the exponent.

Use the following procedure to solve an equation where the variable is in the exponent.

- Write each side of the equation in the same base.
- If necessary, use exponent laws so that each side of the equation contains only one base.
- Equate the exponents on each side of the equation.
- Determine the value of the variable.



Solve the following exponential equations.

a) $5^{2x+3} = 5^7$

$2x+3=7$

$\frac{2x}{2} = \frac{4}{2}$

$\boxed{x=2}$

b) $7^{x-2} = 343$

$7^{x-2} = 7^3$

$x-2=3$

$\boxed{x=5}$

c) $3^{5x-1} = 81^{3x}$

$3^{5x-1} = (3^4)^{3x}$

$3^{5x-1} = 3^{12x}$

$5x-1 = 12x$

$\frac{-1}{7} = \frac{7x}{7}$

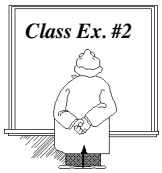
$\boxed{x = -\frac{1}{7}}$

d) $3^x = 27\sqrt{3}$

$3^x = 3^3 \cdot 3^{\frac{1}{2}}$

$3^x = 3^{\frac{7}{2}}$

$\boxed{x = \frac{7}{2}}$



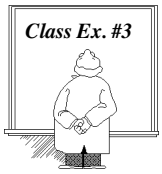
A bacterium triples every six days. The number of bacteria n, present after x days, is given by the formula $n = 3^{\frac{x}{6}}$. After how many days are there 243 bacteria?

$243 = 3^{\frac{x}{6}}$

$3^5 = 3^{\frac{x}{6}}$

$6 \cdot [5 = \frac{x}{6}]$

$x = 30$



Solve the following exponential equations by converting each side to a common base.

a) $27^{x-2} = \dfrac{1}{81^{x+3}}$

$(3^3)^{x-2} = (3^{-4})^{x+3}$

$3^{3x-6} = 3^{-4x-12}$

$3x - 6 = -4x - 12$

$7x = -6$

$x = -\frac{6}{7}$

b) $\left(\dfrac{125}{216}\right)^{\frac{-x}{4}} = \left(\dfrac{6}{5}\right)^{3x-3}$

$\left(\frac{216}{125}\right)^{\frac{x}{4}}$

$\left(\left(\frac{6}{5}\right)^3\right)^{\frac{x}{4}} = \left(\frac{6}{5}\right)^{3x-3}$

$\left(\frac{6}{5}\right)^{\frac{3x}{4}} = \left(\frac{6}{5}\right)^{3x-3}$

$4 \cdot \left[\frac{3x}{4} = 3x - 3\right] \cdot 4$

$3x = 12x - 12$

$\frac{12}{9} = \frac{9x}{9}$

$x = \frac{4}{3}$

Complete Assignment Questions #1 - #12

Assignment

1. Simplify.

a) $49^{x-1} \times 7^{2x-3}$

b) $216^x \div (1296^{5x-4} \times 36^{x+5})$