Exponential Function

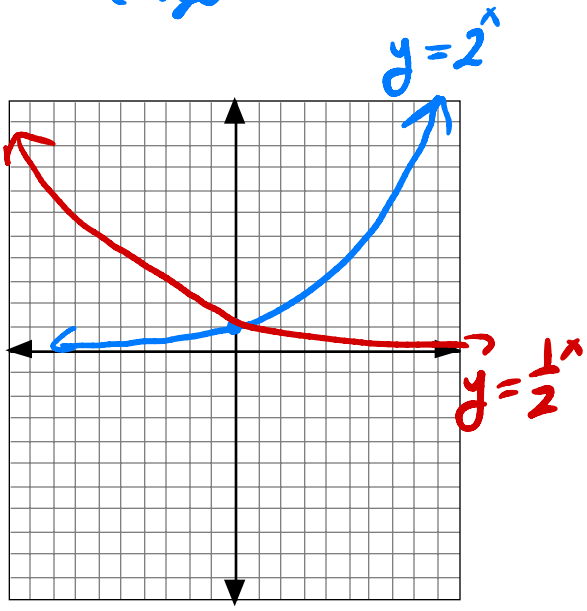
The explorations on the previous page are examples of **exponential functions.** An **exponential function** is a function whose equation is of the form

$y = ab^x$ where $a \neq 0, b > 0, b \neq 1, x \in R$

# of cycles

base rate of change

Comparing the Graphs $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$



a) State the values of a and b for $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$.

b) Sketch the graph of the exponential function with equation $y = 2^x, x \in R$, using the table of values and grid.

as x ↑, y ↑ growth.

x	-3	-2	-1	0	1	2	3	4
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

c) Sketch the graph of the exponential function with equation $y = \left(\frac{1}{2}\right)^x, x \in R$, using the table of values and grid.

as x ↑, y ↓ decay.

x	-3	-2	-1	0	1	2	3	4
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

d) An **asymptote** is a line whose distance from a given curve gets closer and closer to zero. In the above graphs, the x-axis is a horizontal asymptote. Complete the following chart.

Equation of Function	*Domain of Function*	*Range of Function*	*x-intercept of Graph*	*y-intercept of Graph*	*Equation(s) of Asymptotes*
$y = 2^x$	$x = \mathbb{R}$	$y > 0$	none	$(0,1)$	$y = 0$
$y = \left(\frac{1}{2}\right)^x$	$x = \mathbb{R}$	$y > 0$	none	$(0,1)$	$y = 0$

e) Complete the following statements using the words "growth" or "decay".

- $f(x) = 2^x$ is an example of a __growth__ function.
- $f(x) = \left(\frac{1}{2}\right)^x$ is an example of a __decay.__ function.

Exploring the Value of b in $y = ab^x$, where $a = 1$

a) By using a graphing calculator or other technology, sketch the exponential functions with equation:

(i) $y = 3^x$

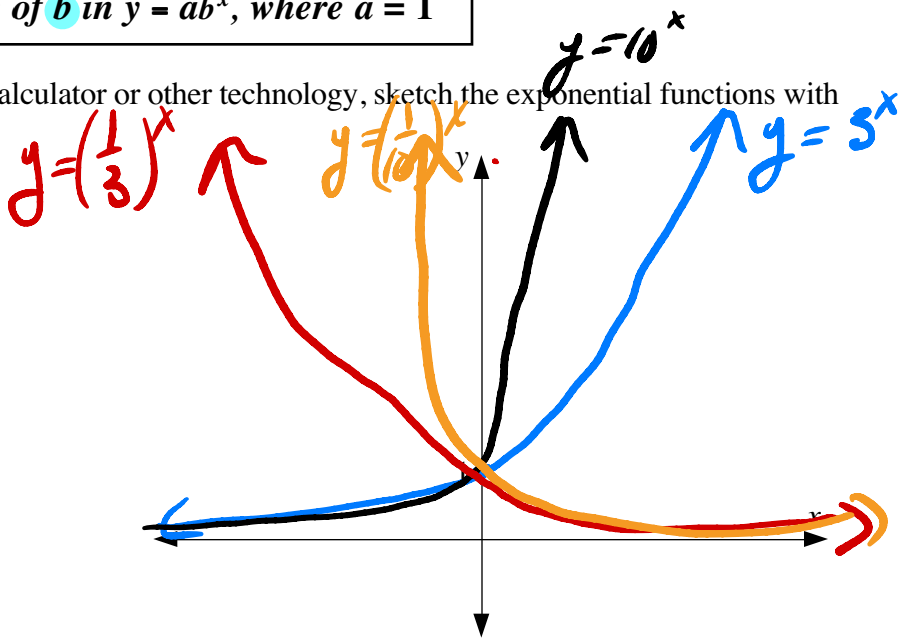
(ii) $y = 10^x$

(iii) $y = \left(\frac{1}{3}\right)^x$

(iv) $y = \left(\frac{1}{10}\right)^x$

growth

decay



b) The value of b affects the steepness of the graph as x increases. Complete the following.

- When $b > 1$, the curve rises more sharply as b increases.
- When $0 < b < 1$, the curve falls more sharply as b decreases.

c) Without using a graphing calculator, make a sketch of the graphs of:

i) $y = 5^x$ **ii)** $y = (0.2)^x$ $y=\left(\frac{1}{5}\right)^x$

d) Verify the solution in c) using a graphing calculator.

e) State the x-intercept for each of the graphs of the form $y = b^x$.

none

f) State the y-intercept for each of the graphs of the form $y = b^x$.

$y = 1$

g) State the domain for each of the graphs of the form $y = b^x$.

$x = \mathbb{R}$

h) State the range for each of the graphs of the form $y = b^x$.

$y > 0$

i) State the equation of the horizontal asymptote for each of the graphs of the form $y = b^x$.

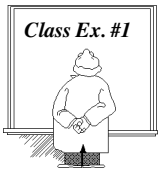
$y = 0$

Characteristics of the Graph of the Exponential Function $f(x) = ab^x$

The following summarizes the basic characteristics of the graph of the exponential function with equation $y = ab^x$.

Use the information from the previous explorations to complete the following.

- The y-intercept is _____ .
- There is _____ x-intercept.
- The x-axis is a _____ _____.
- The domain is _____.
- The range is _____.
- For $a > 0$,
  - When $b > 1$, the function represents a _____ function.
  - When _____, the function represents a decay function .
- The value of b affects the steepness of the graph as x increases.
  - When $b > 1$, the curve _____ sharply as b increases.
  - When $0 < b < 1$, the curve _____ sharply as b decreases.
- The value of a affects the vertical stretch of the graph. Choose the correct alternative.
  - When $a > 1$, the stretch is a(n) (expansion / compression).
  - When $0 < a < 1$, the stretch is a(n) (expansion / compression).
  - When $a < 0$, there is also a reflection in the (x-axis / y-axis).



Describe how the graph of the second function compares to the graph of the first function.

a) $y = 4^x$, $y = 2(4)^{x-2}$

$y \to \frac{1}{2}y$ vert. exp. by a factor of 2
$x \to x-2$ hor. trans. 2 units right

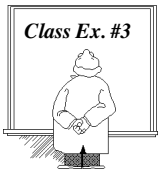
b) $y = 2^x$, $y + 4 = -2^{\frac{x}{5}}$

$y \to y+4$ vert. trans. 4 units down
$y \to -y$ reflection on x-axis
$x \to \frac{1}{5}x$ hor. exp. by a factor of 5.

Explain, using transformations, why the graph of $y = \left(\frac{1}{3}\right)^x$ is a reflection in the y-axis of the graph of $y = 3^x$.

$y = 3^{-x}$

$x \to -x$ refl. on y-axis



Consider the function $f(x) = 4^{x+2} - 6$. Without using a graphing calculator, determine

a) the domain and range of the function

no change → $y > -6$

$x \to x+2$ h.t. 2 units left

$y \to y+6$ v.t. 6 units down.

b) the y-intercept of the graph of the function

let $x = 0$

y-int $= (0, 10)$

c) the equation(s) of any asymptotes of the graph of the function

horizontal asymptote at $y = -6$

Complete Assignment Questions #1 - #11

Assignment

1. State the x and y-intercepts for the graphs of the following:

a) $f(x) = 2^x$ **b)** $f(x) = (2)10^x$ **c)** $f(x) = 2^{10x}$ **d)** $y = \left(-\frac{1}{2}\right)\left(\frac{3}{5}\right)^x$

2. a) State the domain and range of the function $f(x) = ab^x, a, b > 0, x \in R$.

b) Which of the following transformations applied to the graph of $y = ab^x, a, b > 0, x \in R$, would result in a change to the **domain** of the function?

i) horizontal stretch about the y-axis

ii) vertical stretch about the x-axis

iii) horizontal translation

iv) reflection in the x-axis

v) reflection in the y-axis

vi) reflection in the line $y = x$

c) Which of the above transformations applied to the graph of $y = ab^x, a, b > 0, x \in R$, would result in a change to the **range** of the function?