Exponential and Logarithmic Functions Lesson #5: *Evaluating Logarithms*

Review

In the last lesson we compared the graphs of $y = 2^x$ and $y = \log_2 x$.

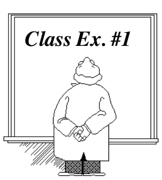
We learned
- the point (3, 8) on the graph of $y = 2^x$ indicates that $8 = 2^3$
- the point (8, 3) on the graph of $y = \log_2 x$ indicates that $3 = \log_2 8$
- the exponential form $8 = 2^3$ and the logarithmic form $3 = \log_2 8$ are equivalent

In this lesson, we will learn how to evaluate logarithms like $\log_3 8$ without reference to a graph or table.

Evaluating Logarithms by Converting to Exponential Form

Kelcie was asked to evaluate $\log_2 8$. Her work is shown below.
Study her work and describe each step on the lines provided.

	$\log_2 8 = v$	Represent $\log_2 8$ with the unknown value "v".
exponential	$2^v = 8$	changed to exponential
	$\underline{2}^v = \underline{2}^3$	rewrite as a common base
	$v = 3$	compare exponents.
logarithmic:	$\log_2 8 = 3$	$v = \log_2 8 = 3$



Evaluate the following logarithms.

a) $\log_3 81 = v$

$3^v = 81$

$\underline{3}^v = \underline{3}^4$

$\boxed{v = 4}$

b) $\log_4 64 = v$

$4^v = 64$

$\underline{4}^v = \underline{4}^3$

$\boxed{v = 3}$

c) $\log_7 7 = v$

$\underline{7}^v = \underline{7}^1$

$\boxed{v = 1}$

d) $\log_{12} 1 = v$

$12^v = 1$

$12^v = 12^0$

$\boxed{v = 0}$

Class Ex. #2

Determine the value of

a) $\log_b 1$

b) $\log_b b$

Class Ex. #3

Evaluate.

a) $2\log_8 512 = v$

$\log_8 512 = \frac{v}{2}$

$8^{\frac{v}{2}} = 512$

$8^{\frac{v}{2}} = 8^3$

$\frac{v}{2} = 3$

$\boxed{v = 6}$

b) $\log_2\left(\frac{1}{32}\right) = v$

$2^v = \frac{1}{32}$

$2^v = 2^{-5}$

$\boxed{v = -5}$

c) $\log_5\sqrt{125} = v$

$5^v = \sqrt{125}$

$5^v = \sqrt{5^3}$

$5^v = 5^{3/2}$

$\boxed{v = \frac{3}{2}}$

Class Ex. #4

a) Evaluate.

i) $5^{\log_5 25}$

$\log_5 25 = 2$

$5^2 = \boxed{25}$

ii) $7^{\log_7 49}$

$\log_7 49 = 2$

$7^2 = \boxed{49}$

iii) $3^{\log_3 27}$

$\log_3 27 = 3$

$3^3 = \boxed{27}$

b) Based on your observations from a) to c), what is the value of $a^{\log_a n}$?
Use inverses to explain your answer.

"n"

exp. & log. are inverse operations. They effectively undo one another.

c) Determine the value of $\log_a a^n$.

$\log_a a^n = $ "n"

Complete Assignment Questions #1 - #3

Evaluating Logarithms by Changing the Base

a) Try to evaluate $\log_5 50$. What problem do you encounter?

$\log_5 50 = v$

b) Choose the correct alternative. The exact value of $\log_5 50$ is

i) between 1.5 and 2 **ii)** between 2 and 2.5 **iii)** between 2.5 and 3

At the moment we are unable to evaluate $\log_5 50$ by converting to exponential form.

We can, however, evaluate the logarithm using a calculator by changing the base to a base the calculator recognizes.

$\log_5 50 = \frac{\log 50}{\log 5}$

To do this, we introduce **Common Logarithms** and **Natural Logarithms**.

LOG * base 10

LN base "e"

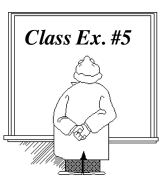
Common Logarithms

Common logarithms are logarithms in base 10, eg. $\log_{10} 1000$.

These logarithms are in such common use that when a base is not given the logarithm is understood to be in base ten. For instance,

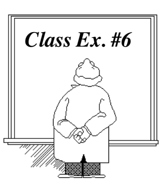
$$\mathbf{\log_{10} 1000 \ \text{ is often written as } \ \log 1000}$$

On a graphing calculator, this can be evaluated using the $\boxed{\text{LOG}}$ key.



Evaluate log 1000 by

a) converting to exponential form **b)** using the $\boxed{\text{LOG}}$ key on a calculator.



In each of the following,

i) estimate the value, to the nearest whole number, without using technology and explain the reasoning

ii) approximate the value to the nearest hundredth using technology

a) $\log_{10} 89$ **b)** log 2000

Investigating the Change of Base Identity

a) i) Evaluate $\log_5 25$ by converting to exponential form.

ii) Evaluate $\dfrac{\log 25}{\log 5}$ and $\dfrac{\ln 25}{\ln 5}$ using a calculator.

iii) Comment on your answers from i) and ii).

b) i) Evaluate $\log_3 243$ by converting to exponential form.

ii) Evaluate $\dfrac{\log 243}{\log 3}$ and $\dfrac{\log_e 243}{\log_e 3}$ using a calculator.

iii) Comment on your answers from i) and ii).

c) In the first page of this lesson, we learned that $\log_2 8 = 3$.
Write $\log_2 8$ in a form which can be evaluated using a calculator and verify the answer.

d) Write $\log_2 64$ in a form which can be evaluated using a calculator.

Change of Base Identity

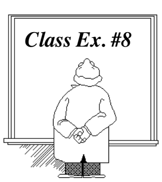
$$\log_b c = \frac{\log_a c}{\log_a b}$$

eg. $\log_5 50 = \dfrac{\log 50}{\log 5}$

We have seen that the above identity is true for converting logarithms to base 10 or base e.
In fact it holds true for converting logarithms to any base. The example below supports this.

i) Evaluate $\log_4 1024$

ii) Evaluate $\dfrac{\log_2 1024}{\log_2 4}$

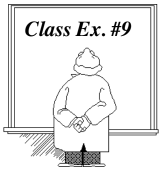


Evaluate the following logarithms to the nearest hundredth by changing the base.

a) $\log_5 221$ $= \dfrac{\log 221}{\log 5} \approx 3.35$

b) $\log_2 \dfrac{1}{1000}$ $\dfrac{\log\left(\frac{1}{1000}\right)}{\log 2} \approx -9.97$

c) $3 \log_7 512$ $3 \cdot \dfrac{\log 512}{\log 7} \approx 9.62$



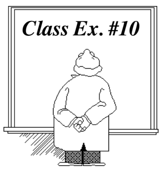
Convert the following logarithms to the base indicated.

a) $\log_6 216$ to base 3

$\frac{\log_3 216}{\log_3 6}$

b) $\log 300$ to base 5

$\frac{\log_5 300}{\log_5 10}$



Find the exact value of the following.

a) $-\log_7\left(\frac{1}{343}\right)$

$-\cancel{\log_7} \cancel{7}^{-3}$

$-(-3) = \boxed{3}$

b) $\cancel{6^{\log_6}} 216$

$= \boxed{216}$

c) $\log_2\sqrt{\frac{1}{1024}}$

$(2^{-10})^{\frac{1}{2}}$

$\cancel{\log_2 2}^{-5} = \boxed{-5}$

d) $\log_7 49^{-5}$

$\log_7 (7^2)^{-5}$

$\cancel{\log_7 7}^{-10} = \boxed{-10}$

Complete Assignment Questions #4- #15

Assignment

$\boxed{\#\ 1\text{-}4, 7, 8, 11, 13}$

1. Evaluate.

a) $\log_{10}1000$ **b)** $\log_{12}144$ **c)** $\log_6 36$ **d)** $\log_{36} 6$

e) $\log_5\sqrt{5}$ **f)** $4 \log_{10} 0.001$ **g)** $\log_2\sqrt{\frac{1}{512}}$ **h)** $-4 \log_8 8^{-4}$

2. State the value of

a) $\log_b 1$ **b)** $\log_c c$ **c)** $\log_x x^z$ **d)** $b^{\log_b 10}$

3. Solve for x.

a) $\log_x 125 = 3$ **b)** $\log_{125} 5 = x$ **c)** $\log_4 x = -8$

4. Evaluate each of the following logarithms.

a) $\log 100$ **b)** $\log 10^6$ **c)** $\log\sqrt{10}$ **d)** $\log 0.01$