Transformations Lesson #1:
Horizontal and Vertical Translations - Part One

Overview

In this unit we will develop an understanding of the effects of **transformations** (operations which move (or map) a figure from an original position to a new position) on the graphs of functions and their related equations. The transformations we will consider are **translations**, **reflections**, **stretches**, and combinations of these.

In particular, we will consider replacements for x and/or y in the function $y = f(x)$ and investigate how the function $y - k = af[b(x - h)] + k$ is related to $y = f(x)$.

Translations

A **translation** is a transformation which slides each point of a figure the same distance in the same direction.

Comparing the Graphs of $y = f(x)$ and $y - k = f(x)$ [or $y = f(x) + k$]

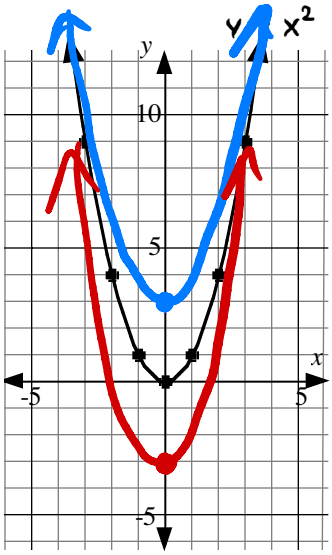
Part 1

a) Complete the table of values. The first one has been completed.

$y = x^2$	
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y - 3 = x^2$	
x	y
-3	12
-2	7
-1	4
0	3
1	4
2	7
3	12

$y + 3 = x^2$	
x	y
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



b) Use the table of values in a) to graph and label each of the functions on the grid.

c) In the second table, y has been replaced by $y - 3$.
What is the effect of this replacement on the graph of $y = x^2$?

vert. trans. up 3 units

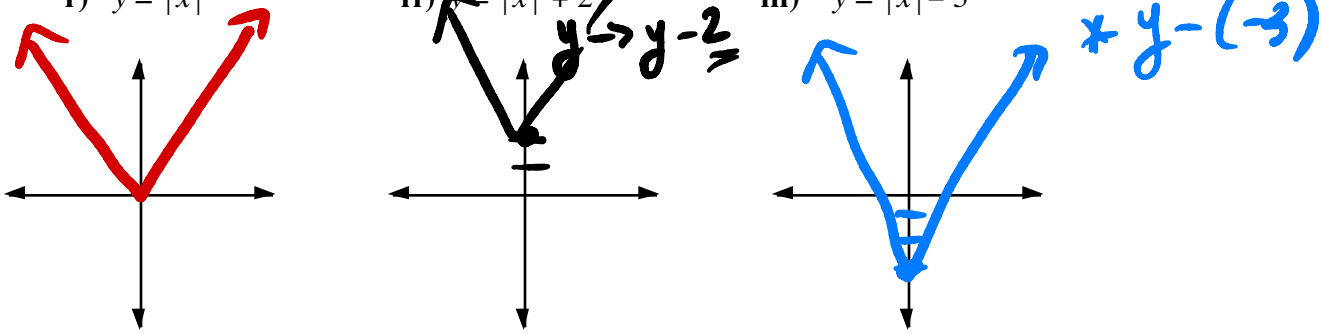
d) In the third table, y has been replaced by $y + 3$.
What is the effect of this replacement on the graph of $y = x^2$?

vert. trans. down 3 units

Part 2

a) Use a graphing calculator to graph the following functions:

i) $y = |x|$ **ii)** $y = |x| + 2$ **iii)** $y = |x| - 3$



b) The equation $y = |x| + 2$ can be rewritten as $y - 2 = |x|$.
How does the replacement of y by $y - 2$ affect the graph of $y = |x|$?

vert. trans. 2 units up

c) The equation $y = |x| - 3$ can be rewritten as $y + 3 = |x|$.
How does the replacement of y by $y + 3$ affect the graph of $y = |x|$?

vert. trans. 3 units down.

d) Using the results of Parts 1 and 2, answer the following questions based on the graph of $y = f(x)$.

i) What is the effect of the **parameter** k on the graph of the function $y = f(x) + k$?

" "

ii) What is the effect of the **parameter** k on the graph of the function $y - k = f(x)$?

vertical trans. by K units up or down

e) Complete the following statements:

- Compared to the graph of $y = f(x)$, the graph of $y - k = f(x)$ results in a vertical translation of k units.
- If $k > 0$, the graph moves up. If $k < 0$, the graph moves down.

The notation $\boldsymbol{y - k = f(x)}$ is often used instead of $y = f(x) + k$ to emphasize that this is a transformation on y. The concept of replacing y by $y - k$ will be very important in this course.

Comparing the Graphs of y = f(x) and y = f(x – h)

horizontal

Part 1

a) Complete the table of values.
The first one has been completed.

h = 3 h = -3 x-(-3)

i) $y = x^2$

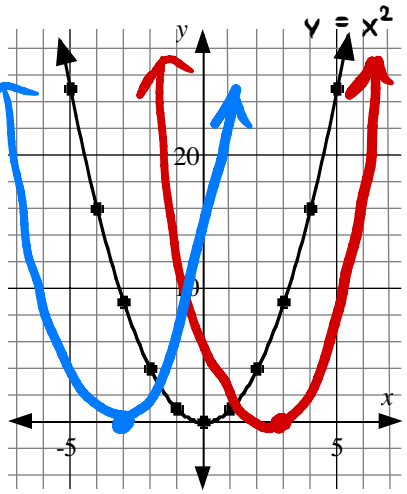
x	y
-4	16
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16

ii) $y = (x - 3)^2$

x	y
-1	16
0	9
1	4
2	1
3	0
4	1
5	4
6	9
7	16

iii) $y = (x + 3)^2$

x	y
-7	16
-6	9
-5	4
-4	1
-3	0
-2	1
-1	4
0	9
1	16



b) Use the table of values in a) to graph and label each of the functions on the same grid.

c) In the second table, x has been replaced by $x - 3$.
What is the effect of this replacement on the graph of $y = x^2$?

hor. trans. 3 units right

d) In the third table, x has been replaced by $x + 3$.
What is the effect of this replacement on the graph of $y = x^2$?

hor trans. 3 units left

Part 2

a) Use a graphing calculator to graph the following functions, observing the effects of replacing x by $x - 4$, and x by $x + 2$.

x -> x-4 h = 4 x -> x+2 h = -2

i) $y = \sqrt{x}$ **ii)** $y = \sqrt{x - 4}$ **iii)** $y = \sqrt{x + 2}$

b) Based on the graph of $y = f(x)$, and using the results of Parts 1 and 2, describe the effect of the **parameter** h on the graph of the function $y = f(x - h)$.

hor. trans. by h units left or right

c) Complete the following statements:

- Compared to the graph of $y = f(x)$, the graph of $y = f(x - h)$ results in a horizontal translation of h units.
- If $h > 0$, the graph moves right. If $h < 0$ the graph moves left.

Replacements For Translations

Given the function $y = f(x)$:

- **replacing y with $y - k$, (*i.e.* $y \to y - k$) describes a vertical translation.**
  $y - k = f(x)$ or $y = f(x) + k$ describes a vertical translation.
- **replacing x with $x - h$, (*i.e.* $x \to x - h$) describes a horizontal translation.**
  $y = f(x - h)$ describes a horizontal translation.

In general, if
$y - k = f(x - h)$
or
$y = f(x - h) + k$ then

$k > 0$ the graph moves up ↑
$k < 0$ the graph moves down ↓
$h > 0$ the graph moves right →
$h < 0$ the graph moves left ←

Class Ex. #1

What happens to the graph of the function $y = f(x)$ if the following changes are made to its equation?

a) replace x with $x + 2$ $h = -2$ h.t. 2 units left

b) replace y with $y - 8$ $K = 8$ v.t. 8 units up.

Class Ex. #2

Describe how the graphs of the following functions relate to the graph of $y = f(x)$.

a) $y = f(x - 3)$ replacement: $x \to x-3$ transformation: h.t. 3 units right

b) $y = f(x) + 4$ $y - 4 = f(x)$ $y \to y-4$ v.t. 4 units up.

c) $y - 1 = f(x + 10)$ $y \to y-1$ v.t. 1 unit up. $x \to x+10$ h.t. 10 units left

---

Class Ex. #3

The point $(2, -3)$ lies on the graph of $y = f(x)$. State the coordinates of the image of this point under the following transformations.

a) $y + 8 = f(x)$ $y \to y+8$ v.t. 8 units down
mapping: $(x, y) \to (x, y-8)$
$(2, -3) \to (2, -11)$

b) $y - 5 = f(x - 7)$ $y \to y-5$ v.t. 5 units up $x \to x-7$ h.t. 7 units right
$(x, y) \to (x+7, y+5)$
$(2, -3) \to (9, 2)$

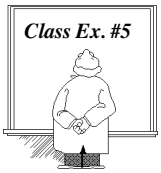
Class Ex. #4

Write the equation of the image of $y = f(x)$ after each transformation.

a) A horizontal translation of 5 units left. $x \to x+5$ $h = -5$
$y = f(x+5)$

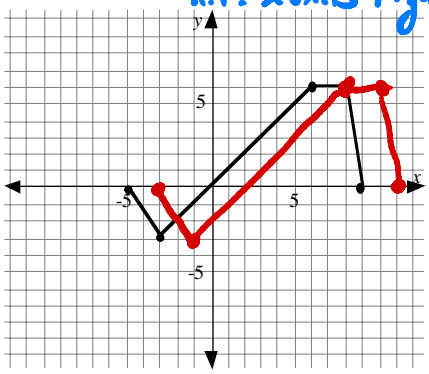
b) A translation of 3 units up. $y \to y-3$ $K = 3$
$y - 3 = f(x)$ or $y = f(x)+3$

c) A translation of m units right and p units down.
$y + p = f(x - m)$
or
$y = f(x - m) - p$

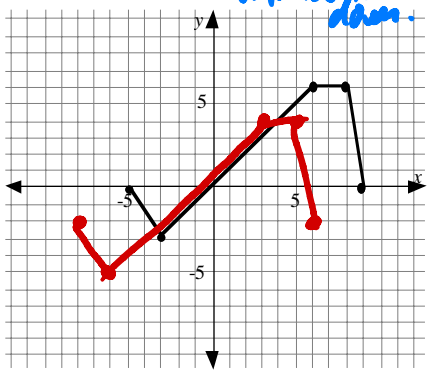


Given the graph of the function $y = f(x)$, sketch the graph of the indicated function.

a) $y = f(x - 2)$



b) $y + 2 = f(x + 3)$



Replacment Notation and Mapping Notation

Do not confuse **mapping notation** with the notation we have used for replacements.

Consider the example where the graph of $y = f(x)$ is transformed to the graph of $y - 2 = f(x - 3)$.

In this example, the replacements for x and y may be written as $x \to x - 3$ and $y \to y - 2$.

Under this transformation, all points on the graph of $y = f(x)$ will move 3 units to the right and 2 units up. The point with coordinates $(4, 6)$ will be translated to the point $(7, 8)$. In general the point with coordinates (x, y) is translated to the point $(x + 3, y + 2)$.

The mapping notation for this translation may be written as $(x, y) \to (x + 3, y + 2)$, implying that the point with coordinates (x, y) is translated to the point $(x + 3, y + 2)$.

Notice that the mapping notation $(x, y) \to (x + 3, y + 2)$, is **NOT** the same as the replacement notation $x \to x + 3$ and $y \to y + 2$.

The mapping notation $(x, y) \to (x + 3, y + 2)$ **is equivalent to** the replacement notation $x \to x - 3$ and $y \to y - 2$.

a) State the coordinates of the image of the point $(-3, 5)$ under the translation described by $(x, y) \to (x - 7, y + 4)$.

b) Write the equation of the image of $y = f(x)$ after the translation $(x, y) \to (x - 6, y + 1)$.

Complete Assignment Questions #1 - #10

Assignment #1 (a,c,e...), 2-10

1. Describe how the graphs of the following functions relate to the graph of $y = f(x)$.

a) $y = f(x + 9)$ **b)** $y = f(x) + 7$ **c)** $y = f(x - 4) + 4$

d) $y - 6 = f(x)$ **e)** $y = 3 + f(x - 5)$ **f)** $y + 2 = f(x + 3) - 10$

2. Write the equation of the image of $y = f(x)$ after each transformation.

a) a vertical translation of 10 units down

b) a horizontal translation of 8 units right and a vertical translation of 9 units up

c) a translation of t units up and s units left

3. The function $y = f(x)$ is transformed to $y = f(x - h) + k$. Find the values of h and k for the following translations.

a) 7 units right **b)** 4 units up and 2 units left **c)** a units right and b units down.

4. The point $(-3, 5)$ lies on the graph of $y = f(x)$. State the coordinates of the image of this point under the following transformations.

a) $y = f(x) + 3$ **b)** $y + 5 = f(x + 2)$ **c)** $(x, y) \rightarrow (x - 7, y - 1)$

5. What happens to the graph of the function $y = f(x)$ if you make these changes to its equation?

a) replace x with $x - 8$ **b)** replace y with $y + 2$

c) replace x with $x + 4$, and y with $y - 7$