Transformations Lesson #3:
Reflections - Part One

Invariant Points

Invariant points are points on a graph which do not move after a transformation.

Comparing the Graphs of $y = f(x)$ and $y = -f(x)$

Part 1

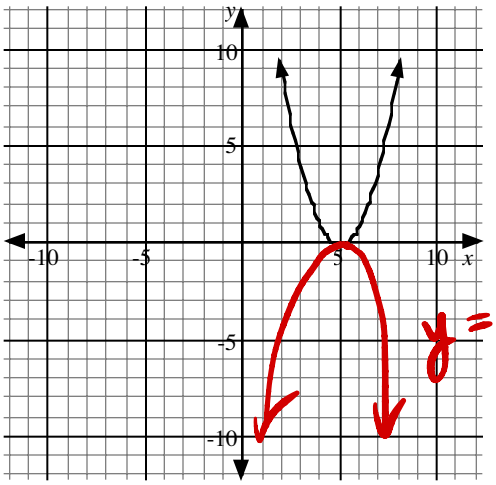
a) The graph of $y = f(x) = x^2 - 10x + 25$ is shown.
Write an equation which represents $y = -f(x)$.

$y \to -y$

$-y = x^2 - 10x + 25$

$y = -x^2 + 10x - 25$

mapping $(x, y) \to (x, -y)$



b) Use a graphing calculator to sketch $y = -f(x)$ and show the graph on the grid.

c) State the coordinates of the invariant point(s).

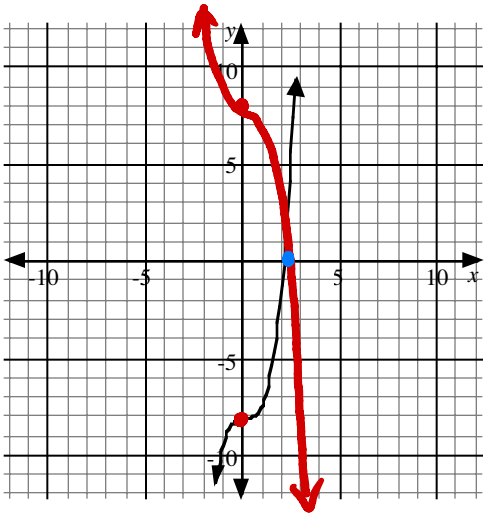
(5, 0)

Part 2

a) The graph of $y = f(x) = x^3 - 8$ is shown.
Write an equation which represents $y = -f(x)$.

$y \to -y$ $\quad -y = x^3 - 8$

$y = -x^3 + 8$



b) Use a graphing calculator to sketch $y = -f(x)$ and show the graph on the grid.

c) State the coordinates of the invariant point(s).

$(x, y) \to (x, -y)$

$(0, -8) \to (0, 8)$

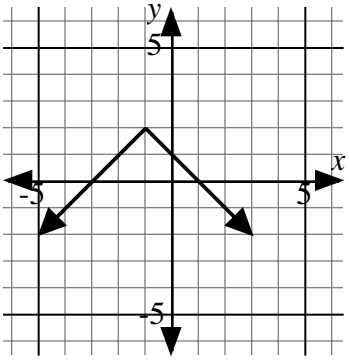
(2, 0)

d) How does the graph of $y = -f(x)$ compare with the graph of $y = f(x)$?

reflection on the x-axis

Part 3

The graph of $y = f(x)$ is shown.
Sketch the graph of $y = -f(x)$.



If we replace y with $-y$, then $y = f(x)$ becomes $-y = f(x)$,
which is equivalent to $y = -f(x)$.
So the replacement in this example is $y \to -y$.

Comparing the Graphs of y = f(x) and y = f(–x)

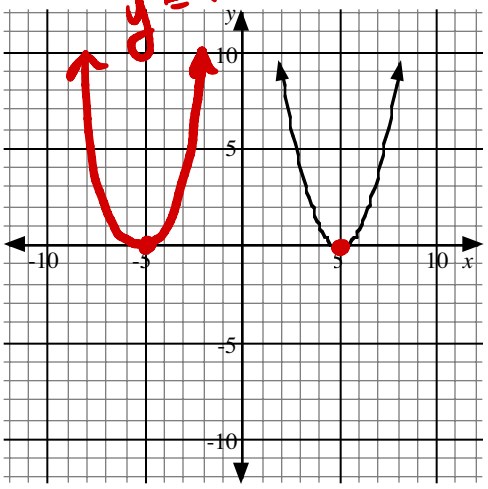
x → –x

mapping
(x, y) → (–x, y)
(5, 0) → (–5, 0)

Part 1

a) The graph of $y = f(x) = x^2 - 10x + 25$ is shown.
Write an equation which represents $y = f(-x)$.

$y = (-x)^2 - 10(-x) + 25$
$y = x^2 + 10x + 25$



b) Use a graphing calculator to sketch $y = f(-x)$ and show the graph on the grid.

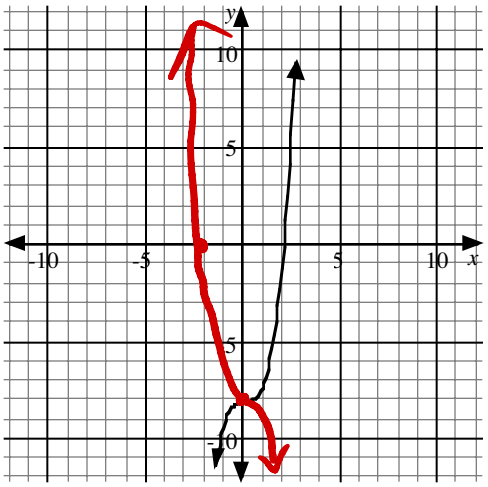
c) State the coordinates of the invariant point(s).

(0, 25)
y-int.

Part 2

a) The graph of $y = f(x) = x^3 - 8$ is shown.
Write an equation which represents $y = f(-x)$.

$y = (-x)^3 - 8$
$y = -x^3 - 8$



b) Use a graphing calculator to sketch $y = f(-x)$ and show the graph on the grid.

c) State the coordinates of the invariant point(s).

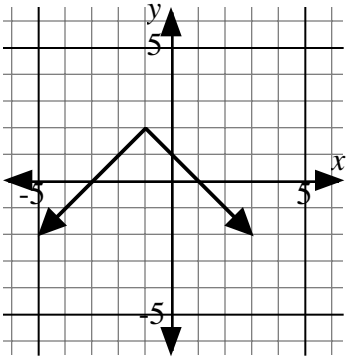
(0, –8)

d) How does the graph of $y = f(-x)$ compare with the graph of $y = f(x)$?

reflection on the y-axis

Part 3

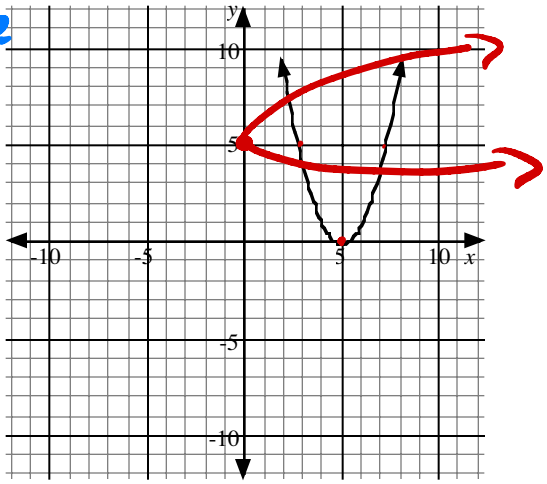
The graph of $y = f(x)$ is shown.
Sketch the graph of $y = f(-x)$.



If we replace x with $-x$, then $y = f(x)$ becomes $y = f(-x)$.
So the replacement in this example is $x \rightarrow -x$.

Comparing the Graphs of $y = f(x)$ and $y = f^{-1}(x)$ or $x = f(y)$

Part 1

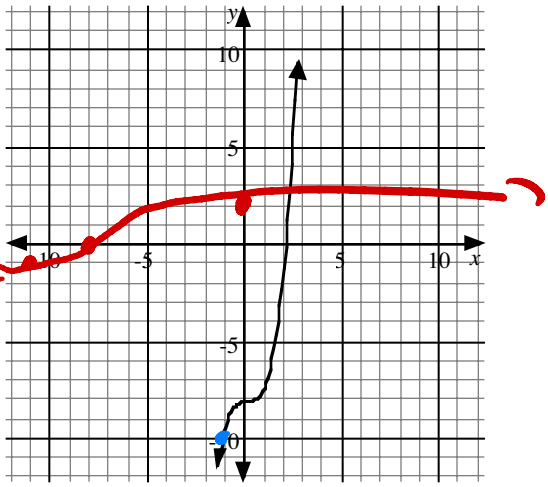


a) The graph of $y = f(x) = (x - 5)^2$ is shown.
Write an equation which represents $x = f(y)$ and solve for y.

b) Use a graphing calculator to sketch $x = f(y)$ and show the graph on the grid.

c) Although there are four points of intersection of the graphs, explain why there are only two invariant points. Mark the invariant points on the grid.

Part 2



a) The graph of $y = f(x) = x^3 - 8$ is shown.
Write an equation which represents $y = f^{-1}(x)$.

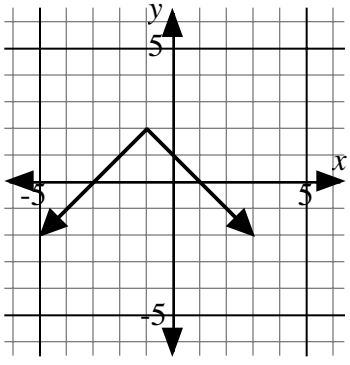
b) Use a graphing calculator to sketch $y = f^{-1}(x)$ and show the graph on the grid.

c) Mark the invariant point(s) on the grid.

d) How does the graph of $x = f(y)$ compare with the graph of $y = f(x)$?

Part 3

The graph of $y = f(x)$ is shown.
Sketch the graph of $x = f(y)$.

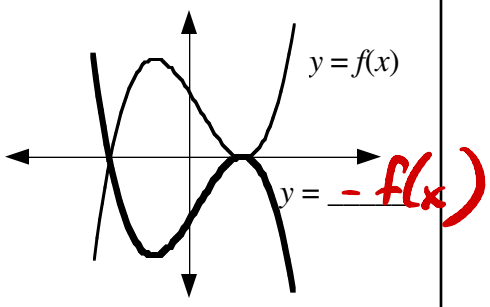
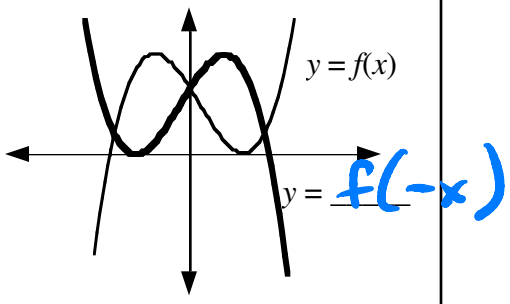
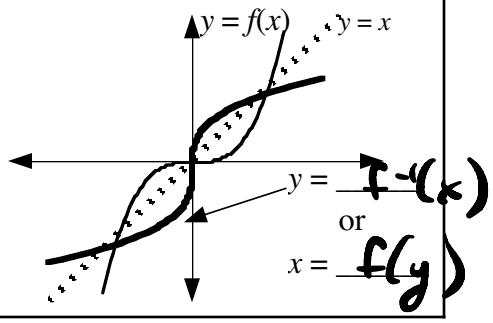


If we replace x with y, and y with x, then $y = f(x)$ becomes $x = f(y)$.
So the replacements in this example are $x \to y$ and $y \to x$.

Reflections

A **reflection** is a transformation which reflects (or flips) a figure about a line.

Fill in the following blanks which summarize the previous investigations.

Reflection	*Function*	*Graph*
Reflection in the *x*-axis	If the graph of $y = f(x)$ is reflected in the x-axis, then it is the graph of $y = -f(x)$.	
Reflection in the *y*-axis	If the graph of $y = f(x)$ is reflected in the y-axis, then it is the graph of $y = f(-x)$.	
Reflection in the line $y = x$	If the graph of $y = f(x)$ is reflected in the line $y = x$, then it is the graph of $x = f(y)$ or $y = f^{-1}(x)$.	

Given the function $y = f(x)$:

- **replacing x with $-x$, (*i.e.* $x \to -x$) describes a reflection in the *y*-axis.**
  $y = f(-x)$ describes a reflection in the *y*-axis.

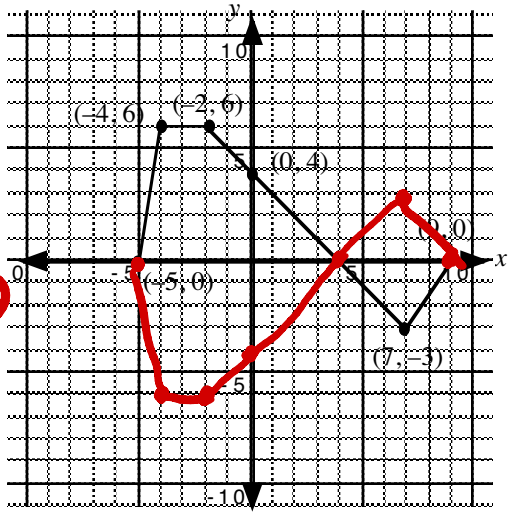
- **replacing y with $-y$, (*i.e.* $y \to -y$) describes a reflection in the *x*-axis.**
  $-y = f(x)$ or $y = -f(x)$ describes a reflection in the *x*-axis.

- **interchanging x and y, (*i.e.* $x \to y$, $y \to x$) describes a reflection in the line $y = x$.**
  $x = f(y)$ or $y = f^{-1}(x)$ describes a reflection in the line $y = x$.

Class Ex. #1

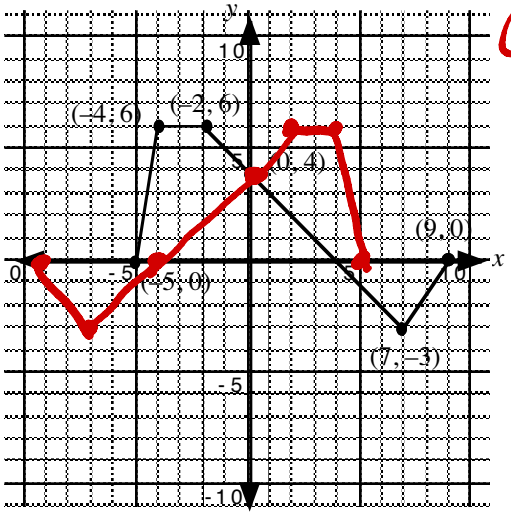
The graph of $y = f(x)$ is shown. Sketch:

a) $y = -f(x)$



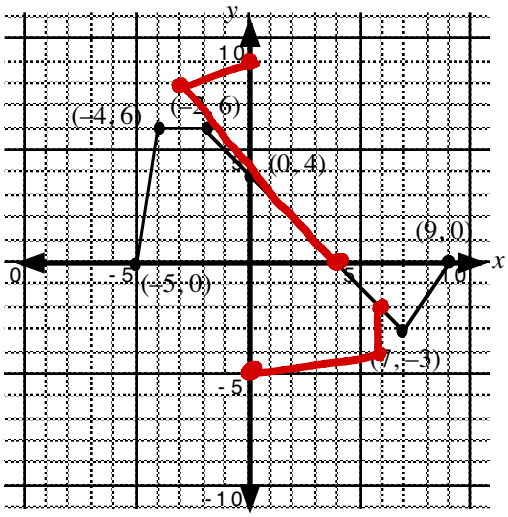
y → −y
reflection in the x-axis
(x, y) → (x, −y)

b) $y = f(-x)$



x → −x
ref. on the y-axis
(x, y)
(−x, y)

c) $x = f(y)$

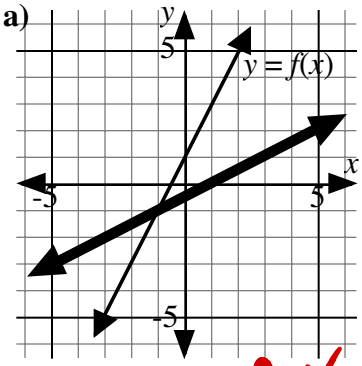


x ↔ y
ref. on y = x
(x, y) → (y, x)

Class Ex. #2

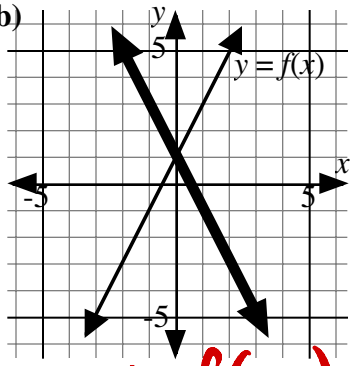
The graph drawn in the thick line is a reflection of the graph drawn in the thin line. Write an equation for each graph drawn in the thick line.

a)



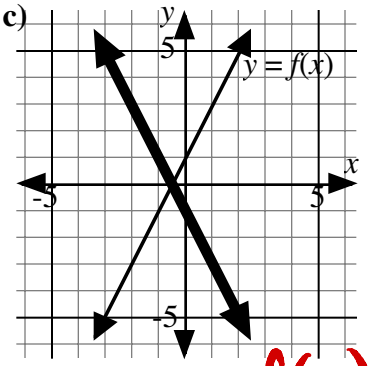
$y = f^{-1}(x)$

b)



$y = f(-x)$

c)



$y = -f(x)$

Complete Assignment Questions #1 - #6

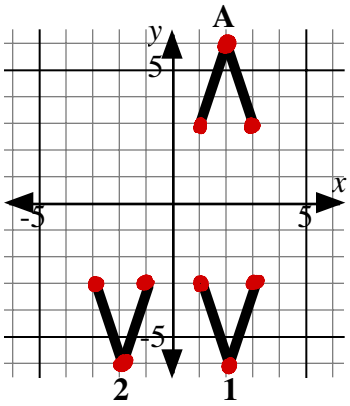
Combining Reflections

Part 1 - Transforming $y = f(x)$ to $y = -f(-x)$

$x \to -x$
$y \to -y$

The table below shows how to "build" $y = -f(-x)$ from $y = f(x)$.

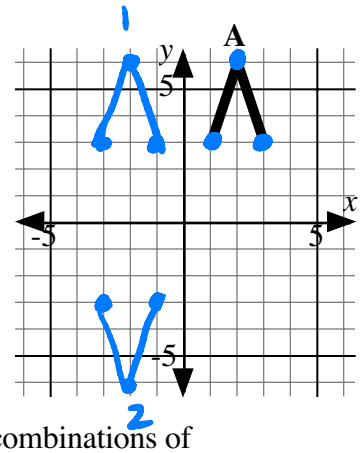
Transformation	Replacement for x or y	Current Equation
1. reflection in the x-axis	$y \to -y$	$-y = f(x)$ $y = -f(x)$
2. reflection in the y-axis	$x \to -x$	$y = -f(-x)$



The transformations in the table are applied to shape **A** in the order shown.
The images are shown on the grid.

a) Complete the table below to determine the equation which results from changing the order in which the reflections are carried out.

Transformation	Replacement for x or y	Current Equation
1. reflection in the y-axis	$x \to -x$	$y = f(-x)$
2. reflection in the x-axis	$y \to -y$	$-y = f(-x)$ $y = -f(-x)$



b) On the grid above, sketch the image of shape **A** under the combinations of transformations in a).

c) Does the order in which the reflections are carried out affect the final image?

NO b/c they are affecting different variables

d) Describe two sets of transformations, in order, which can be applied to the graph of $y = f(x)$ to produce the graph of $y = -f(-x)$.

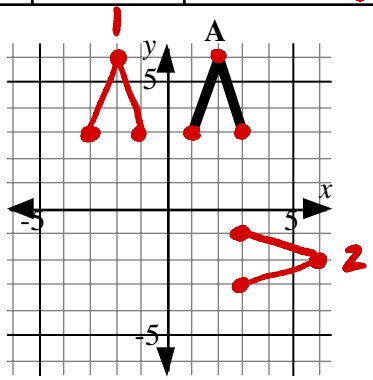
① refl. on the x-axis's then refl. on y-axis's

② refl. on y-axis's then refl. on x-axis's

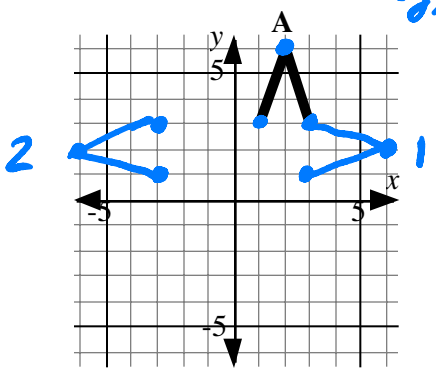
Part 2 - Transforming $y = f(x)$ to $x = f(-y)$ and $x = -f(y)$

In each case, complete the table and sketch the combination of transformations on the grid.

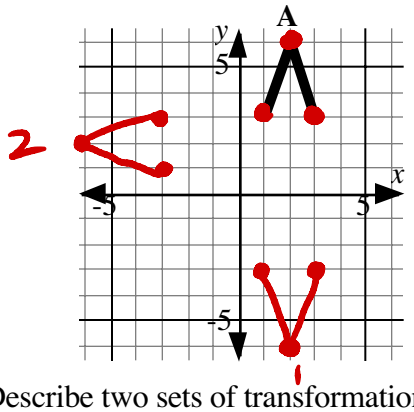
Transformation	Replacement for x or y	Current Equation
1. reflection in the y-axis	$x \to -x$	$y = f(-x)$
2. reflection in $y = x$	$y \leftrightarrow x$	$x = f(-y)$



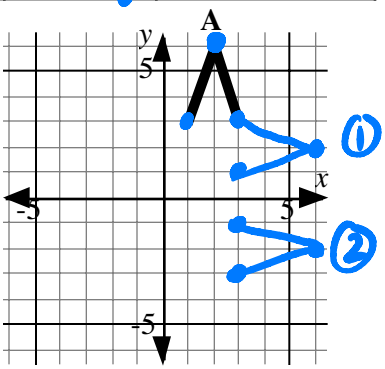
Transformation	Replacement for x or y	Current Equation
1. reflection in $y = x$	$y \leftrightarrow x$	$x = f(y)$
2. reflection in the y-axis	$x \to -x$	$-x = f(y)$ $x = -f(y)$



Transformation	Replacement for x or y	Current Equation
1. reflection in the x-axis	$y \to -y$	$-y = f(x)$ $y = -f(x)$
2. reflection in $y = x$	$y \leftrightarrow x$	$x = -f(y)$



Transformation	Replacement for x or y	Current Equation
1. reflection in $y = x$	$y \leftrightarrow x$	$x = f(y)$
2. reflection in the x-axis	$y \to -y$	$x = f(-y)$



a) Describe two sets of transformations, in order, which can be applied to the graph of $y = f(x)$ to produce the graph of $x = f(-y)$.

b) Describe two sets of transformations, in order, which can be applied to the graph of $y = f(x)$ to produce the graph of $x = -f(y)$.

order matters b/c x & y are being swapped.

Complete Assignment Questions #7 - #14

#1-13