Transformations Lesson #5:
Stretches About the x- or y- axis - Part One

> ### *Comparing the Graphs of y = f(x) and y = af(x), where a > 0*

Part 1

$y \to \frac{1}{3}y$

The graph of $y = f(x) = \sqrt{4 - x^2}$ is shown.

a) Write an equation which represents $y = 3f(x)$.

$\frac{1}{3}y = \sqrt{4-x^2}$

$y = 3\sqrt{4-x^2}$

vertical expansion by a factor of 3

$(x, y) \to (x, 3y)$

b) Use a graphing calculator to sketch $y = 3f(x)$ on the grid.

c) Describe how the number 3 in $y = 3f(x)$ affects:
- the general sketch of $y = f(x)$

  vertical expansion by a factor of 3

- the x-intercepts of the graph of $y = f(x)$

  none (b/c y = 0)

- the y-intercept of the graph of $y = f(x)$.

  multiplied by 3

Part 2

$y \to 2y$
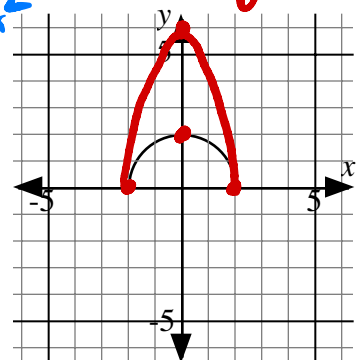
vertical compression by a factor of $\frac{1}{2}$

$(x, y) \to (x, \frac{y}{2})$

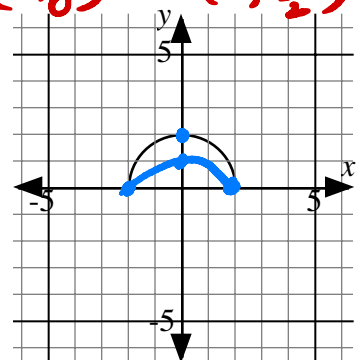**a)** Write an equation which represents $y = \frac{1}{2}f(x)$.

b) Use a graphing calculator to sketch $y = \frac{1}{2}f(x)$ on the grid.

$y \to 2y$

c) Describe how the number $\frac{1}{2}$ in $y = \frac{1}{2}f(x)$ affects:
- the general sketch of $y = f(x)$

  vertical compression by a factor of $\frac{1}{2}$

- the x-intercepts of the graph of $y = f(x)$

  none (b/c y = 0)

- the y-intercept of the graph of $y = f(x)$.

  multiplied by $\frac{1}{2}$

d) Complete the following statement using the results of Parts 1 and 2.

Compared to the graph of $y = f(x)$, the graph of $y = af(x)$
results in a _____ stretch about the ___-axis by a factor of _____ .

Note
If we replace y with $\frac{1}{a}y$, then $y = f(x)$ becomes $\frac{1}{a}y = f(x)$, which is equivalent to $y = af(x)$.

So the replacement in this example is $y \to \frac{1}{a}y$.

Comparing the Graphs of y = f(x) and y = f(bx), where b > 0

Part 1

$x \rightarrow 4x$

The graph of $y = f(x) = \sqrt{4 - x^2}$ is shown.

a) Write an equation which represents $y = f(4x)$.
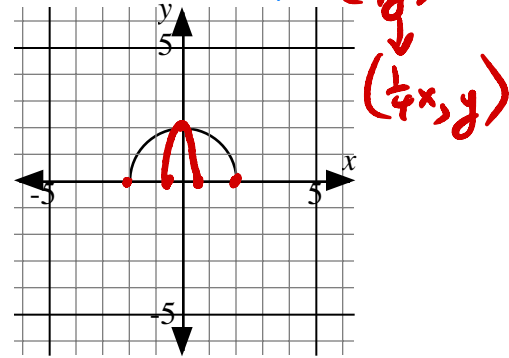
$$y = \sqrt{4 - (4x)^2}$$

$$y = \sqrt{4 - 16x^2}$$

horizontal compression by a factor of $\frac{1}{4}$

(x, y)

$\downarrow$

$(\frac{1}{4}x, y)$

b) Use a graphing calculator to sketch $y = f(4x)$ on the grid.

c) Describe how the number 4 in $y = f(4x)$ affects:

- the general sketch of $y = f(x)$

- the x-intercepts of the graph of $y = f(x)$

  multiplied by $\frac{1}{4}$

- the y-intercept of the graph of $y = f(x)$.

  none (b/c x = 0)

Part 2

$x \rightarrow \frac{1}{3}x$

hor. exp. by a factor of 3

$(x, y) \rightarrow (3x, y)$

a) Write an equation which represents $y = f\left(\frac{1}{3}x\right)$.

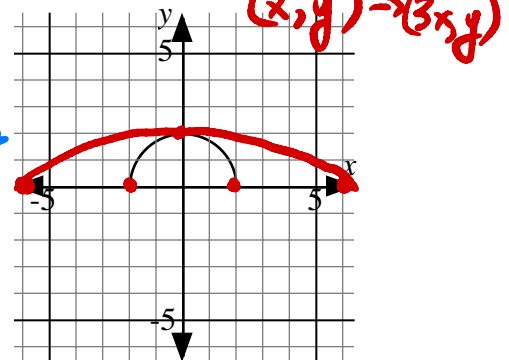$$y = \sqrt{4 - \left(\frac{1}{3}x\right)^2}$$

$$y = \sqrt{4 - \frac{1}{9}x^2}$$

b) Use a graphing calculator to sketch $y = f\left(\frac{1}{3}x\right)$ on the grid.

c) Describe how the number $\frac{1}{3}$ in $y = f\left(\frac{1}{3}x\right)$ affects:

- the general sketch of $y = f(x)$

- the x-intercepts of the graph of $y = f(x)$

  multiplied by 3

- the y-intercept of the graph of $y = f(x)$.

  none (b/c x = 0)

d) Complete the following statement using the results of Parts 1 and 2.

Compared to the graph of $y = f(x)$, the graph of $y = f(bx)$

results in a _____ stretch about the ___-axis by a factor of $\frac{1}{b}$.

Note If we replace x with bx, then $y = f(x)$ becomes $y = f(bx)$.
So the replacement in this example is $x \rightarrow bx$.

Stretches

In mathematics we use the word **stretch** to represent both an expansion or a compression. In this course we only consider stretches about the *x*- and *y*-axis. *reciprocal already taken*

In the table below, the graph of $y = f(x)$ and the graph of $y = af(x)$ or $y = f(bx)$ is given. Fill in the blanks.

a or b	*Horizontal or Vertical Stretch*	*Graph*
$0 < a < 1$ *vert. compression*	The graph of $y = f(x)$ will be stretched __vertically__ by a factor of _a_ about the _X_-axis.	$y = f(x)$ $y = af(x)$
$a > 1$ *vert. expansion*	The graph of $y = f(x)$ will be stretched __vertically__ by a factor of _a_ about the _X_-axis.	$y = f(x)$ $y = af(x)$
$a < 0$	The graph of $y = f(x)$ will be reflected in the __X-axis__ and stretched vertically about the _X_-axis.	$y = f(x)$ $y = af(x)$
$0 < b < 1$ *hor. expansion*	The graph of $y = f(x)$ will be stretched __horizontally__ by a factor of $\frac{1}{b}$ about the _y_-axis.	$y = f(x)$ $y = af(x)$
$b > 1$ *hor. compression*	The graph of $y = f(x)$ will be stretched __horizontally__ by a factor of $\frac{1}{b}$ about the _y_-axis.	$y = f(x)$ $y = af(bx)$
$b < 0$	The graph of $y = f(x)$ will be reflected in the __y-axis__ and stretched horizontally about the _y_-axis.	$y = f(x)$ $y = af(x)$

Note $y = af(x)$ can be written as $\dfrac{1}{a}y = f(x)$.

Given the function $y = f(x)$:

• **replacing x with bx, $(i.e.\, x \rightarrow bx)$ describes a horizontal stretch about the y-axis.**
   i.e. $y = f(bx)$ describes a horizontal stretch.

• **replacing y with $\dfrac{1}{a}y$, $(i.e.\;\; y \rightarrow \dfrac{1}{a}y)$ describes a vertical stretch about the x-axis.**

   i.e. $\dfrac{1}{a}y = f(x)$ or $y = af(x)$ describes a vertical stretch.

In general, if $\dfrac{1}{a}y = f(bx)$ or $y = af(bx)$, then for

> $a > 0$ - vertical stretch about the x-axis by a factor of a
>
> $a < 0$ - vertical stretch about the x-axis by a factor of $|a|$
>   and a reflection in the x-axis
>
> $b > 0$ - horizontal stretch about the y-axis by a factor of $\dfrac{1}{b}$
>
> $b < 0$ - horizontal stretch about the y-axis a by a factor of $\dfrac{1}{|b|}$
>
> and a reflection in the y-axis.


Class Ex. #1   Write the replacement for x or y and write the equation of the image of $y = f(x)$ after each transformation.

a) a horizontal stretch by a factor of 6 about the y-axis $x \rightarrow \tfrac{1}{6}x$

$$y = f(\tfrac{1}{6}x)$$

b) a vertical stretch by a factor of $\tfrac{1}{5}$ about the x-axis $y \rightarrow 5y$

$$5y = f(x)$$
$$y = \tfrac{1}{5}f(x)$$

c) a reflection in the x-axis and a vertical stretch about the x-axis by a factor of 3

$$-\tfrac{1}{3}y = f(x)$$ $y \rightarrow -y$ $y \rightarrow \tfrac{1}{3}y$
$$y = -3f(x)$$ $y \rightarrow -\tfrac{1}{3}y$

d) a horizontal stretch about the y-axis by a factor of $\tfrac{1}{2}$ and

a vertical stretch about the x-axis by a factor of $\tfrac{1}{4}$ $x \rightarrow 2x$

$$4y = f(2x)$$ $y \rightarrow 4y$
$$y = \tfrac{1}{4}f(2x)$$
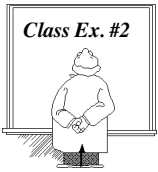
Class Ex. #2 How does the graph of $3y = f(x)$ compare with the graph of $y = f(x)$?

$y \rightarrow 3y$

vert. comp. by a factor of $\frac{1}{3}$
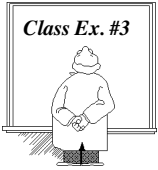
Class Ex. #3 What happens to the graph of the function $y = f(x)$ if you make these changes?

a) Replace x with $4x$.

hor. comp. by a factor of $\frac{1}{4}$

b) Replace y with $\frac{1}{3}y$.

vert. exp. by a factor of 3
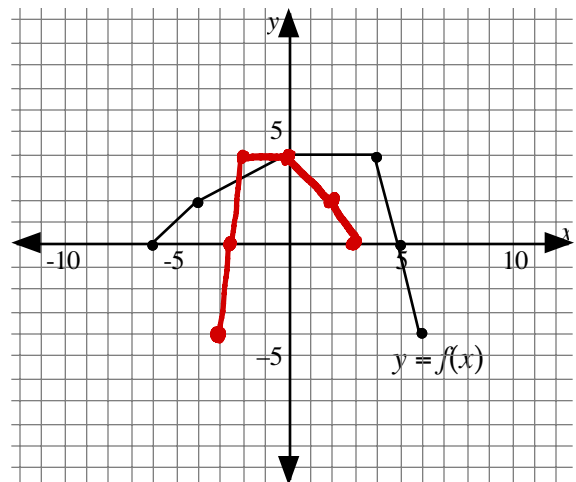
c) Replace y with $6y$ and x with $\frac{1}{3}x$.

vert. comp. by a factor of $\frac{1}{6}$
hor. exp. by a factor of 3

Class Ex. #4 The graph of $y = f(x)$ is shown.

Sketch $y = f(-2x)$.

$x \rightarrow -2x$

reflection on the y-axis
hor comp. by a factor of $\frac{1}{2}$

$(x, y) \longrightarrow (-\frac{1}{2}x, y)$

#1-6

$y = f(x)$

Complete Assignment Questions #1 - #7