

Transformations Lesson #6: Stretches about the x- or y-axis - Part Two

Review

Given the function $y = f(x)$:

- replacing x with bx , (i.e. $x \rightarrow bx$) describes a horizontal stretch about the y-axis.
i.e. $y = f(bx)$ describes a horizontal stretch.

- replacing y with $\frac{1}{a}y$, (i.e. $y \rightarrow \frac{1}{a}y$) describes a vertical stretch about the x-axis.
i.e. $\frac{1}{a}y = f(x)$ or $y = af(x)$ describes a vertical stretch.

In general, if $\frac{1}{a}y = f(bx)$ or $y = af(bx)$, then for:

$a > 0$ - vertical stretch about the x-axis by a factor of a

$a < 0$ - vertical stretch about the x-axis by a factor of $|a|$
and a reflection in the x-axis

$b > 0$ - horizontal stretch about the y-axis by a factor of $\frac{1}{b}$

$b < 0$ - horizontal stretch about the y-axis a by a factor of $\frac{1}{|b|}$
and a reflection in the y-axis.



Write the equation of the image of

- a) $y = x^2$ after a horizontal stretch about the y-axis by a factor of $\frac{3}{4}$

$x \rightarrow \frac{4}{3}x$ $y = \left(\frac{4}{3}x\right)^2 = \frac{16}{9}x^2 = y$

factor reciprocal replacement

- b) $y = \sqrt{x} - 3$ after a horizontal stretch by a factor of 4 about the y-axis and a vertical stretch by a factor of 2 about the x-axis

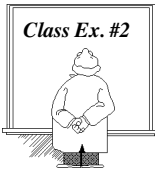
$\frac{1}{2}y = \sqrt{\frac{1}{4}x} - 3$
 $2 \cdot \left[\frac{1}{2}y = \frac{1}{2}\sqrt{x} - 3 \right]$ $y = \sqrt{x} - 6$

$x \rightarrow \frac{1}{4}x$
 $y \rightarrow \frac{1}{2}y$

- c) $y = 3x + 7$ after a vertical stretch about the x-axis by a factor of $\frac{1}{3}$ and a reflection in the x-axis.

$-3y = 3x + 7$
 $\frac{-3y}{-3} = \frac{3x}{-3} + \frac{7}{-3}$
 $y = -x - \frac{7}{3}$

$y \rightarrow 3y$
 $y \rightarrow -y$



Describe how the graph of the second function compares to the graph of the first function.

a) $y = f(x), y = f\left(\frac{1}{2}x\right)$

$x \rightarrow \frac{1}{2}x$
hor. stretch by a factor of 2

b) $y = 2^x, y = 2^{3x}$

$x \rightarrow 3x$
hor comp. by a factor of $\frac{1}{3}$

c) $y = |x|, y = -2|x|$

$y \rightarrow -\frac{1}{2}y$
vert. exp by a factor of 2
refl. on the x-axis

d) $y = x^3, 3y = x^3$

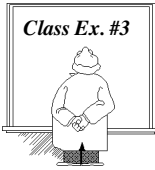
$y \rightarrow 3y$
vert. comp. by a factor of $\frac{1}{3}$

e) $y = |x|, y = 2\left|\frac{1}{3}x\right|$

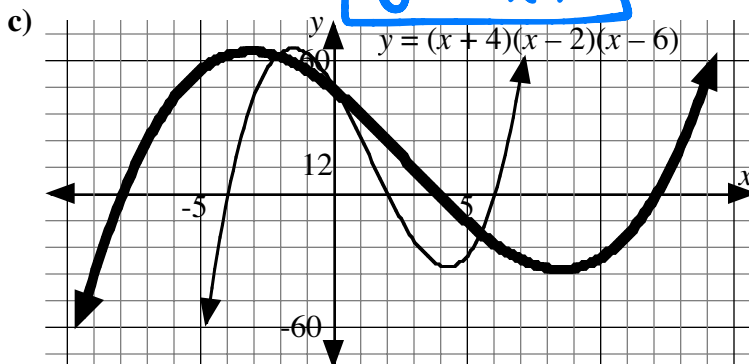
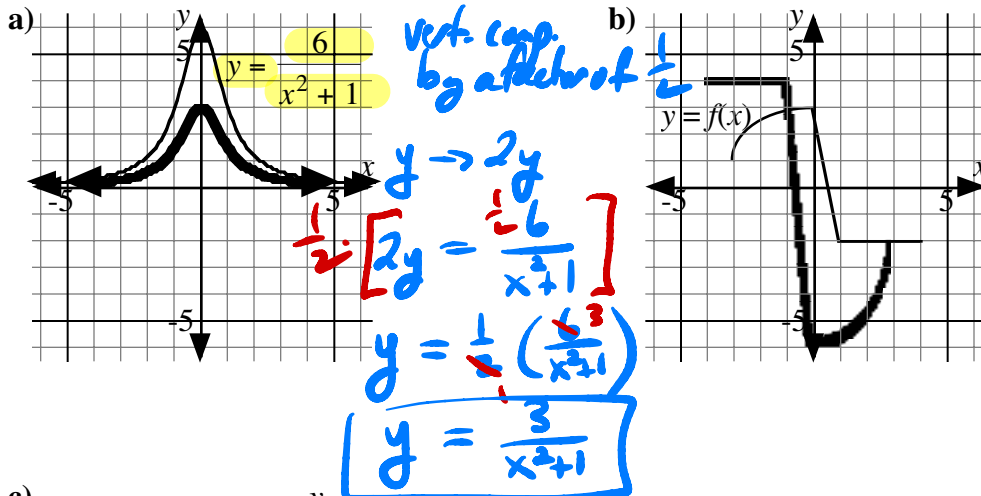
$y \rightarrow \frac{1}{2}y$ $x \rightarrow \frac{1}{3}x$
vert. exp by a factor of 2 hor. exp by a factor of 3

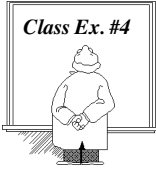
f) $y = |x|, y = |-2x|$

$x \rightarrow -2x$
hor comp. by a factor of $\frac{1}{2}$
reflection on y-axis



The function represented by the thick line is a stretch of the function represented by the thin line. Write an equation for each function represented by the thick line.





Class Ex. #4

A polynomial function has the equation $P(x) = (x - 4)(x + 3)(x + 6)$. Determine the zeros and the y -intercept if the following transformations are applied.

a) $y = -3P(x)$

b) $y = P\left(-\frac{1}{2}x\right)$

Complete Assignment Questions #1 - #6

Assignment

#1-3

1. Write the equation of the image of

a) $y = |x + 1|$ after a vertical stretch about the x -axis by a factor of $\frac{7}{9}$

b) $y = 2^x$ after a horizontal stretch by a factor of 3 about the y -axis

c) $y = \sqrt{x - 2}$ after a vertical stretch about the x -axis by a factor of 4 and a reflection in the x -axis

d) $y = \sqrt{x - 2}$ after a horizontal stretch about the y -axis by a factor of 4 and a reflection in the y -axis

e) $y = \sin x^\circ$ after a horizontal stretch about the y -axis by a factor of $\frac{3}{4}$ and a vertical stretch about the x -axis by a factor of $\frac{1}{2}$