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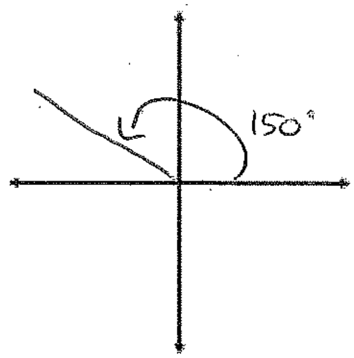
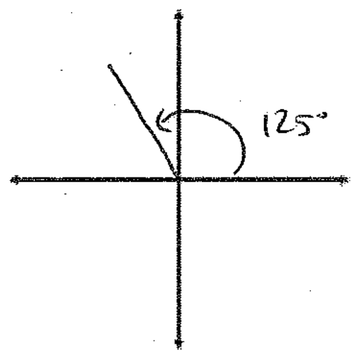
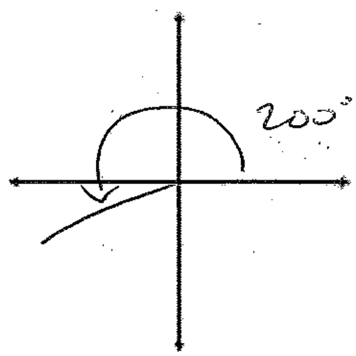
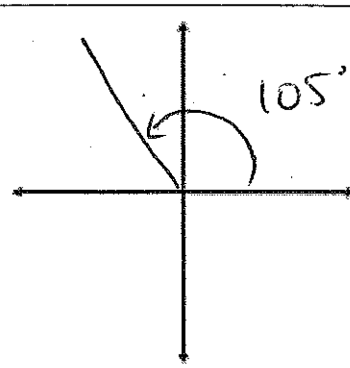
Trigonometry Review
Lessons 1 – 4 (v13a)

1. Complete the following table.

Reference Angle	Quadrant	Sketch	Rotation Angle
30°	2		150°
55°	2		125°
20°	3		200°
75°	2		105°

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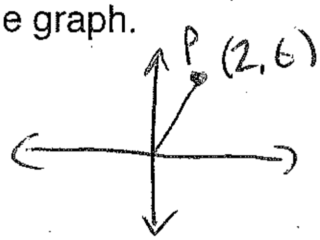
2. The point P(2, 6) lies on the terminal arm of an angle θ.

   a. Sketch the graph.



   b. Calculate the value of *r*.

$$r^2 = x^2 + y^2$$
$$= 2^2 + 6^2$$
$$= 40$$

$$r = \sqrt{40}$$
$$\boxed{r = 2\sqrt{10}}$$

   c. Determine the exact values of the following with rational denominators.

      i. $\sin\theta = \dfrac{y}{r} = \dfrac{6}{2\sqrt{10}} = \dfrac{6\sqrt{10}}{2(10)} = \boxed{\dfrac{3\sqrt{10}}{10}}$

      ii. $\cos\theta = \dfrac{x}{r} = \dfrac{2}{2\sqrt{10}} = \dfrac{2\sqrt{10}}{2(10)} = \boxed{\dfrac{\sqrt{10}}{10}}$

      iii. $\tan\theta. = \dfrac{y}{x} = \dfrac{6}{2} = \boxed{3}$

3. Rewrite the following as the same trigonometric function of a positive acute angle.

   a. sin 342°

S	A
T	C

ref ∠ = 18°
Quadrant 4 → sin −ve
$\boxed{-\sin 18°}$

   b. tan 238°

S	A
T	C

ref ∠ = 58°
Quadrant 2 → tan +ve
$\boxed{\tan 58°}$

4. Complete the following statement:
   Cosine ratios have *negative* values in quadrants __2__ and __3__.

S	A
T	C

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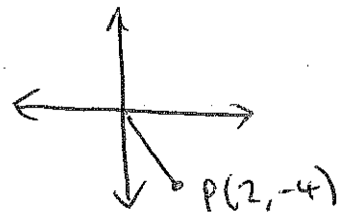
5. The point P(2, -4) lies on the terminal arm of an angle θ in standard position.

   a. Sketch the graph.



   b. Determine the exact values of the following with rational denominators.

$$r^2 = x^2 + y^2 \qquad r = \sqrt{20} = 2\sqrt{5}$$
$$= (2)^2 + (-4)^2 = 20$$

i. $\sin\theta = \frac{y}{r} = \frac{-4}{2\sqrt{5}} = \frac{-4\sqrt{5}}{2(5)} = \boxed{\frac{-2\sqrt{5}}{5}}$

ii. $\cos\theta = \frac{x}{r} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

iii. $\tan\theta = \frac{y}{x} = \frac{-4}{2} = \boxed{-2}$

6. If $\tan\theta = -\frac{12}{5}$ and $\cos\theta$ is positive, then find the exact value of $\sin\theta$ and $\cos\theta$.

$\tan\theta = \frac{y}{x} = -\frac{12}{5}$ → either x or y is negative, we need to determine which

$\cos\theta$ is positive, so $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$ Quadrant 2 and y -ve,

$\therefore \; y = -12$

$x = 5$

$$r^2 = x^2 + y^2$$
$$= (5)^2 + (-12)^2$$
$$= 169$$
$$r = \sqrt{169} = 13$$

$$\boxed{\begin{array}{l} \sin\theta = \frac{y}{r} = \frac{-12}{13} \\ \cos\theta = \frac{x}{r} = \frac{5}{13} \end{array}}$$

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7. Find the measure of θ, to the nearest degree, where $0° \le \theta \le 360°$.

a. $\cos\theta = -0.9511$

S | A
T | C

ref ∠ $= \cos^{-1}(0.9511)$
$= 18°$

$\cos\theta$ -ve so
Quadrant 2, 3

$\boxed{\theta = 162°, \ = 198°}$

b. $\tan\theta = 0.4695$

ref ∠ $= \tan^{-1}(0.4695)$

S | A
T | C

$= 25°$

$\tan\theta$ +ve, so
Quadrant 1, 3

$\boxed{\theta = 25°, \ = 205°}$

c. $\sin\theta = -0.9063$

ref $\theta = \sin^{-1}(0.9063)$

S | A
T | C

$= 65°$

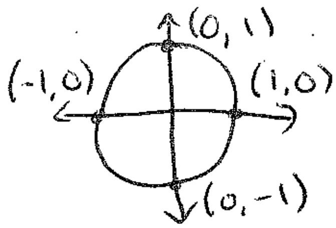
$\sin\theta$ -ve, so
Quadrant 3, 4

$\boxed{\theta = 245°, \ = 295°}$

8. Find the measure of θ, to the nearest degree, where $180° \le \theta \le 360°$.

a. $\sin\theta = 0$

From unit circle

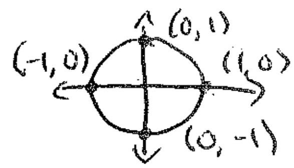


In unit circle, $\sin\theta = \frac{y}{r} = \frac{y}{1} = y$

$\therefore$ $\boxed{\theta = 180°, \ = 360°}$

b. $\cos\theta = -1$

From unit circle,



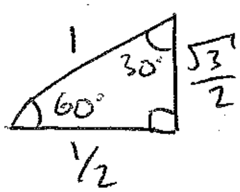
In unit circle, $\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\therefore$ $\boxed{\theta = 180°}$

9. Solve the equation $2\cos\theta - 3 = -2$ where $180° \le \theta \le 360°$.

$$2\cos\theta - 3 = -2$$
$$2\cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$

From



$\cos\theta = \frac{x}{r}$

ref ∠ $= 60°$

$\therefore$ $\boxed{\theta = 300°}$

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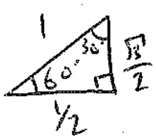
Answer the following questions without your calculator. Use the special triangles or the unit circle to find the answers.

10. Find the exact value of the following with rational denominators.

a. cos 300°

ref ∠ = 60°

Quadrant 4, cos +ve



$\cos 300° = \cos 60°$

$\boxed{= \frac{1}{2}}$

b. tan 210°

ref ∠ = 30°

Quadrant 3, tan +ve

$\tan 210° = \tan 30°$

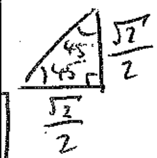
$= \frac{\sin 30°}{\cos 30°}$

$= \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

c. sin 135°

ref ∠ = 45°

Quadrant 2, sin +ve



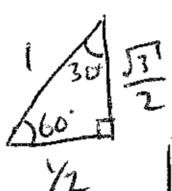
$\sin 135° = \sin 45°$

$= \boxed{\frac{\sqrt{2}}{2}}$

11. Find the measure of θ, to the nearest degree, where $0° \le \theta \le 360°$.

a. $\cos\theta = \frac{1}{2}$

cos θ +ve,
so Quadrant 1, 4

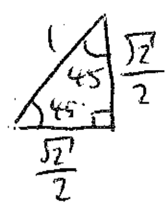


ref ∠ = 60° from special triangle

$\boxed{\theta = 60°, = 300°}$

b. $\sin\theta = -\frac{\sqrt{2}}{2}$

sin −ve,
so Quadrant 3, 4

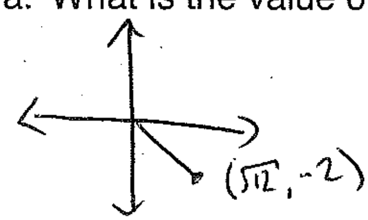


ref ∠ = 45° from special triangle

$\boxed{\theta = 225°, = 315°}$

12. The point $(\sqrt{12}, -2)$ is on the terminal arm of an angle θ.

a. What is the value of tan θ ?



$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{x} = \frac{-2}{\sqrt{12}} = \frac{-2}{2\sqrt{3}}$

$= \boxed{\frac{-\sqrt{3}}{3}}$

b. What is the value of θ ?

ref ∠ $= \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = 30°$

Quadrant 4, $\boxed{\theta = 330°}$

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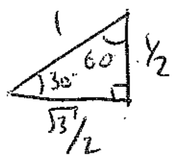
13. Find the exact value of the following.

a. $\sin 150°$

S | A
T | C

Q2 → sin +ve

$\sin 150° = \sin 30°$

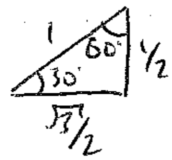


$= \boxed{1/2}$

b. $\cos 210°$

Q3 → cos −ve

$\cos 210° = -\cos 30°$

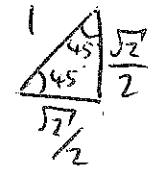


$= \boxed{-\frac{\sqrt{3}}{2}}$

c. $\tan 315°$

Q4 → tan −ve

$\tan 315° = -\tan 45°$



$= -\frac{\sin 45°}{\cos 45°}$

$= -\frac{\sqrt{2}/2}{\sqrt{2}/2}$

$= \boxed{-1}$

14. Find the measure of θ, to the nearest degree, where $0° \leq \theta \leq 360°$.

a. $\sin\theta = -\frac{1}{2}$ sin −ve, Q3, 4

S | A
T | C

From special triangle or $\sin^{-1}$ on calculator,

ref ∠ = 30°

$\boxed{\theta = 210°, = 330°}$

b. $\cos\theta = \frac{\sqrt{3}}{2}$ cos +ve, Q1, 4

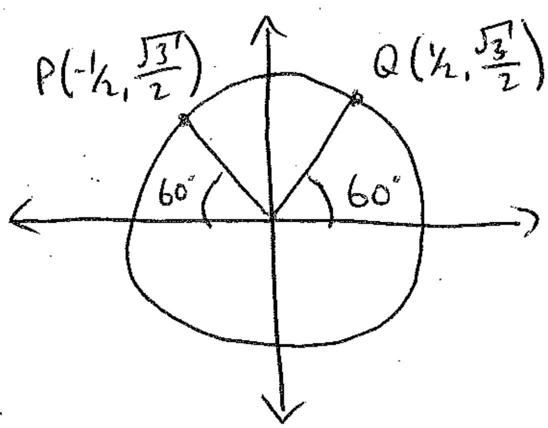
S | A
T | C

From special triangle or $\cos^{-1}$ on calculator,

ref ∠ = 30°

$\boxed{\theta = 30°, = 330°}$

15. $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are two points on the unit circle. If an object rotates counterclockwise from point P to point Q, through what angle has it rotated?



To rotate from P to Q you must rotate

$180° + 60 + 60 = 300°$

counterclockwise

$\boxed{\text{Angle} = 300°}$