MATH 12 PROBLEM SET

TRIGONOMETRIC IDENTITIES & EQUATIONS

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FINAL SCORE: 20/20

BE SURE TO SHOW ALL YOUR WORK

UNLESS OTHERWISE STATED, FINAL ANSWERS MUST BE EXACT (NO DECIMALS) AND IN LOWEST TERMS

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1. Which one of the following is the exact ratio of $\sin 285^\circ$? ref ∠ = 75° in quad 4   1. A

a) $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ b) $\frac{-\sqrt{3}+1}{2\sqrt{2}}$ c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin -45 \cos -30 + \cos -45 \sin -30$

$\left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) =$

2. If $\sin\theta > 0$ and $\sec\theta = -\frac{\sqrt{7}}{2}$, then which one of the following is the exact ratio of $\sin 2\theta$?   2. C

$2\sin\theta\cos\theta$ $y = \sqrt{3}$ $\cos\theta = \frac{-2}{\sqrt{7}}$ $\sin\theta = \frac{\sqrt{3}}{\sqrt{7}}$

a) $\frac{2\sqrt{3}}{\sqrt{7}}$ b) $\frac{\sqrt{3}}{\sqrt{7}}$ c) $-\frac{4\sqrt{3}}{7}$ d) $\frac{4\sqrt{3}}{7}$

$2\left(\frac{\sqrt{3}}{\sqrt{7}}\right)\left(\frac{-2}{\sqrt{7}}\right)$

Numerical Response

3. Rounded to the nearest hundredth, what is the *largest* possible solution for the equation $\sec^2 x = x - \sin 3x$ if $0 \le x < 2\pi$?   3. 5.18

from graphing calculator

Numerical Response

4. To verify the identity $\sin x \tan x + \sec x = \frac{\sin^2 x + 1}{\cos x}$, Allie substituted 1.4 rad for x on each side. Rounded to the nearest tenth, what numerical value will each side of this identity be equal to?   4. $\boxed{11.6}$

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5. **Use the statement** $\dfrac{\tan\theta+\sin\theta}{1+\sec\theta}=\sin\theta$ **to answer the following questions.** **(4 marks)**

a) Verify that this statement is true if $\theta=\dfrac{\pi}{4}$.

1

$$\frac{\tan\frac{\pi}{4}+\sin\frac{\pi}{4}}{1+\sec\frac{\pi}{4}}=\sin\frac{\pi}{4}$$

$$\frac{1+\frac{1}{\sqrt2}}{1+\sqrt2}=\frac{1}{\sqrt2}$$

$$\frac{\frac{\sqrt2+1}{\sqrt2}}{1+\sqrt2} \quad \boxed{\frac{1}{\sqrt2}=\frac{1}{\sqrt2}}$$

b) Algebraically prove that this statement is an identity.

2

$$\frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{1+\frac{1}{\cos\theta}}=\sin\theta$$

$$\frac{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}{\frac{\cos\theta+1}{\cos\theta}}$$

$$\frac{\sin\theta(1+\cos\theta)}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta+1} \quad \boxed{\sin\theta=\sin\theta}$$

c) For what *radian* value(s) of x is this statement undefined?

1

wherever $\sec\theta=-1$ ∴ where $\cos\theta=-1$

$\boxed{\theta\neq\pi}$ & $\tan\theta$ cannot be undefined @ $\boxed{\frac{\pi}{2},\ \frac{3\pi}{2}}$

$\frac{\pi}{2}+\pi n$

$\pi+2\pi n.$

6. **Algebraically prove each of the following trigonometric identities.** **(4 marks)**

a) $\tan^2\theta+1=\dfrac{2}{1+\cos2\theta}$

$$\sec^2\theta=\frac{2}{1+\cos^2\theta-\sin^2\theta}$$

$$\frac{1}{\cos^2\theta}=\frac{2}{\cos^2\theta+\cos^2\theta}$$

$$\frac{1}{\cos^2\theta}=\frac{\cancel{2}}{\cancel{2}\cos^2\theta}$$

$$\frac{1}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

b) $\sin\left(\dfrac{\pi}{3}+\theta\right)+\cos\left(-\dfrac{\pi}{6}-\theta\right)=\sqrt3\cos\theta$

$$\sin\tfrac{\pi}{3}\cos\theta+\cos\tfrac{\pi}{3}\sin\theta+\cos\tfrac{-\pi}{6}\cos\theta+\sin\tfrac{-\pi}{6}\sin\theta=\sqrt3\cos\theta$$

$$\frac{\sqrt3}{2}\cos\theta+\cancel{\tfrac{1}{2}\sin\theta}+\frac{\sqrt3}{2}\cos\theta+\cancel{\tfrac{-1}{2}\sin\theta}=\sqrt3\cos\theta$$

$$\frac{\cancel{2}\sqrt3\cos\theta}{\cancel{2}}$$

$$\sqrt3\cos\theta=\sqrt3\cos\theta$$

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7. **Algebraically solve the equation $3\tan^2\theta + \tan\theta - 2 = 0$ for θ, where $0^\circ \le \theta < 360^\circ$. Round your final answers to the nearest degree.** (3 marks)

let $n = \tan\theta$

$3n^2 + n - 2 = 0$

$(3n-2)(n+1) = 0$

$n = \frac{2}{3}$ $\quad n = -1$

$\tan\theta = \frac{2}{3}$ $\quad \tan\theta = -1$

$\theta = -45^\circ + 360 = \boxed{315^\circ}$

$\tan^{-1}\left(\frac{2}{3}\right) = \boxed{34^\circ}$ $\quad \theta_2 = 180 + (-45) = \boxed{135^\circ}$

$\theta_2 = 180 + 34^\circ$

$= \boxed{214^\circ}$

$\boxed{34^\circ, 135^\circ, 214^\circ, 315^\circ}$

8. **Algebraically determine the general *radian* solution of the equation $2\sin\frac{1}{3}\theta = \sqrt{3}$.** (2 marks)

$\sin\frac{1}{3}\theta = \frac{\sqrt{3}}{2}$ $\quad$ period $= \frac{2\pi}{\frac{1}{3}}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{3}\theta$ $\quad = 6\pi$

$\frac{\pi}{3} = \frac{1}{3}\theta$

$\boxed{\pi} = \theta$

$\theta_2 = \pi - \pi$

$= \boxed{0}$

general solution $= 0 + 6\pi n$ (π or 2π)

$= \pi + 6\pi n$

9. **Algebraically solve the equation $\sin 2\theta = \sin\theta$, where $0 \le \theta \le 2\pi$.** (3 marks)

include 2π but not necessary *

$2\sin\theta\cos\theta = \sin\theta$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ $\quad \cos\theta = \frac{1}{2}$

$\theta = \boxed{0, \pi, 2\pi}$ $\quad \theta = \boxed{\frac{\pi}{3}}$

$2\pi - \frac{\pi}{3}$

$= \boxed{\frac{5\pi}{3}}$

$\boxed{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi}$

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